JET

Journal of ENERGY TECHNOLOGY

University of Maribor Press

VOLUME 18
ISSUE 3
NOVEMBER 2025

JET

Editor-in-Chief Jurij Avsec
(University of Maribor, Slovenia)

Co-editors Amor Chowdhury
(University of Maribor, Slovenia)

Brigita Ferčec
(University of Maribor, Slovenia)

Matej Fike
(University of Maribor, Slovenia)

Miralem Hadžiselimović
(University of Maribor, Slovenia)

Gorazd Hren
(University of Maribor, Slovenia)

Zdravko Praunseis
(University of Maribor, Slovenia)

Sebastijan Seme
(University of Maribor, Slovenia)

Bojan Štumberger
(University of Maribor, Slovenia)

Peter Virtič
(University of Maribor, Slovenia)

Technical Editors Jan Perša
(University of Maribor, University of Maribor Press, Slovenia)

Sonja Krajnc
(University of Maribor, Slovenia)

Lecturer Shelagh Margaret Hedges

Cover designer Jan Perša
(University of Maribor, University Press, Slovenia)

Cover graphics photo: Jurij Avsec, 2025

Graphics material Sources are own unless otherwise noted.

Publication type E-publication

EDITORIAL OFFICE ADDRESS
University of Maribor
Faculty of Energy Technology
Hočevarjev trg 1, 8270 Krško, Slovenia
Koroška cesta 62a, 3320 Velenje, Slovenia
e-mail: jet@um.si
https://www.fe.um.si

PUBLISHED BY
University of Maribor
University Press
Slomškov trg 15, 2000 Maribor, Slovenia
e-mail: zalozba@um.si, zalozba.journals@um.si
https://press.um.si/, https://journals.um.si

University of Maribor

Faculty of Energy Technology

ISSN 2463-7955 (electronic)
ISSN 2630-2535 (printed)

The **Journal of Energy Technology** is a peer-reviewed journal published in May, August, November and December by University of Maribor Press and issued by University of Maribor, Faculty of Energy Technology (Slovenia). The journal is indexed and abstracted in databases INSPEC©, Proquest and EBSCO. For more information, please visit our website at https://journals.um.si/index.php/jet

University of Maribor Press

Slovenian Research and Innovation Agency

Journal of Energy Technology is co-funded by Slovenian Research and Innovation Agency

Journal of Energy Technology

Volume 18 Number 3 November 2025

JOURNAL OF ENERGY TECHNOLOGY
Vol. 18, No. 3, November 2025

Editorial

JURIJ AVSEC
Editor-in-Chief

In a world hungry for available energy to meet its economic needs, the energy mix is diverse – and despite the growth of renewables, fossil fuels still form the core of global energy consumption. According to the IEA, hydrocarbon energy – coal, oil and natural gas (58 %) – still dominated in 2023. The share of global used energy has increased by 53 % in the last 23 years.

The situation is similar in electricity production. However, the structure of global electricity production is changing rapidly. In 2023, renewable sources – hydro, wind, solar, biomass and others – together contributed about a third of all the electricity generated, while nuclear energy remained stable at around 9 %. Coal remained the largest single source of electricity (35 %). The amount of electricity in the world has increased by 94 % in the last 23 years.

These changes reflect the beginning of a shift towards a more sustainable model – the growth of renewable and nuclear energy at the expense of fossil fuels means reduced emissions of harmful greenhouse gases, and – hopefully – a more sustainable future. At the same time, the growth in energy demand, especially in developing and emerging economies, shows that we will need strong energy strategies: a combination of more clean energy, increased energy efficiency and careful use of resources.

This raises the question: are we ready to make decisive decisions? If we want to stay within the goals of climate neutrality while ensuring energy security, the future will require prudent decisions – and the courage to change.

Uvodnik

V svetu, ki hrepeni po razpoložljivi energiji za svoje gospodarske potrebe, ima raznolik energijski miks ključno vlogo – kljub rasti obnovljivih virov fosilna goriva še vedno predstavljajo jedro globalne porabe energije. Po podatkih IEA za leto 2023 energija iz ogljikovodikov – premoga, nafte in zemeljskega plina – še vedno prevladuje (58 %). V zadnjih 23 letih se je količina uporabljene energije v svetu povečala za 53 %.

Podobno velja za proizvodnjo električne energije, čeprav se njena struktura hitro spreminja. V letu 2023 so obnovljivi viri – voda, veter, sonce, biomasa in drugi – skupaj prispevali približno tretjino vse proizvedene električne energije, medtem ko jedrska energija ostaja stabilna pri približno 9 %. Premog ostaja največji posamezni vir elektrike (35 %). V zadnjih 23 letih se je količina proizvedene električne energije na svetovni ravni povečala za 94 %.

Te spremembe odražajo začetek prehoda k bolj trajnostnemu energetskemu modelu – rast obnovljivih virov in jedrske energije na račun fosilnih goriv pomeni zmanjšanje emisij toplogrednih plinov in – upajmo – bolj trajnostno prihodnost. Hkrati pa rast povpraševanja po energiji, zlasti v državah v razvoju in hitro rastočih gospodarstvih, kaže, da bomo potrebovali močne energetske strategije: kombinacijo več čiste energije, večje energetske učinkovitosti in premišljene rabe virov.

Na tem mestu se pojavi pomembno vprašanje: ali smo pripravljeni na odločilne korake? Če želimo ostati znotraj ciljev podnebne nevtralnosti in hkrati zagotoviti energijsko varnost, bo prihodnost zahtevala preudarne odločitve – in pogum za spremembe.

JOURNAL OF ENERGY TECHNOLOGY
Vol. 18, No. 3, pp. 145–160, November 2025

Original Scientific Article

SYSTEMATIC DERIVATION OF CLARKE AND PARK TRANSFORMATIONS THROUGH VECTOR REPRESENTATION IN THREE-PHASE SYSTEMS

FILIP FILIPOVIĆ, MILUTIN PETRONIJEVIĆ, NEBOJŠA MITROVIĆ, BOJAN BANKOVIĆ, ANĐELA STOJILJKOVIĆ, VOJKAN KOSTIĆ
University of Niš, Faculty of Electronic Engineering, Niš, Serbia
filip.filipovic@elfak.ni.ac.rs, milutin.petronijevic@elfak.ni.ac.rs, nebojsa.mitrovic@elfak.ni.ac.rs, bojan.bankovic@elfak.ni.ac.rs, andjela.stojiljkovic@elfak.ni.ac.rs, vojkan.kostic@elfak.ni.ac.rs

CORRESPONDING AUTHOR
filip.filipovic@elfak.ni.ac.rs

Submitted
4. 11. 2025

Accepted
20. 11. 2025

Published
28. 11. 2025

Abstract This paper presents a comprehensive pedagogical treatment of Clarke and Park transformations through systematic vector representation, designed specifically for educational purposes in power electronics and motor control courses. While these fundamental transformations are ubiquitous in modern applications–from field-oriented control of AC machines to grid-connected converter control–educational materials often present them either as matrix operations without geometric foundation, or embedded within machine-specific derivations that obscure the general mathematical structure. We address this pedagogical need by progressing systematically from three-phase voltage equations in the time domain to a spatial vector representation in three-dimensional space, deriving the Clarke transformation through explicit geometric projection onto the plane where balanced quantities reside, and, subsequently, deriving the Park transformation as a time-varying rotation of the Clarke frame. The work establishes an amplitude-invariant formulation, provides an explicit geometric interpretation of the 35.26° angle between coordinate systems, and includes worked numerical examples demonstrating complete transformations. This unified pedagogical treatment bridges the gap between practical application and mathematical foundations, providing educators with a complete geometrically intuitive framework suitable for graduate-level instruction and professional development.

Keywords
Clarke transformation, Park transformation, coordinate transformations, power electronics education, motor control, reference frame theory

https://doi.org/10.18690/jet.18.3.145-160.2025

University of Maribor Press

1 Introducton

Three-phase electrical systems constitute the foundation of the modern electrical power infrastructure, from generation and transmission to motor drives and power electronic converters. The analysis, control, and optimization of these systems depend fundamentally on the mathematical transformations that convert voltages, currents, and flux linkages between different reference frames. Among these coordinate transformations, the Clarke transformation ($abc \rightarrow \alpha\beta 0$) and Park transformation ($\alpha\beta 0 \rightarrow dq0$) have emerged as indispensable analytical tools since their introduction in the early 20th century.

1.1 Historical Development and Significance

The theoretical foundations of three-phase coordinate transformations date back to the pioneering work of Robert Park in 1929, who introduced what became known as Park's transformation for analyzing synchronous machines. Park's groundbreaking insight was to transform the stator quantities of a synchronous machine to a reference frame rotating with the rotor, thereby turning time-varying inductances into constants and simplifying the machine's equations. This transformation enabled analytical solutions to problems that were previously intractable in AC machine analysis, and laid the groundwork for modern control theory applications in electrical machines.

Edith Clarke contributed by developing what is now called the Clarke transformation, published in her work on symmetrical components and related topics. Clarke's transformation offered an intermediate step that converts three-phase quantities from the *abc* natural reference frame to a stationary two-axis orthogonal reference frame ($\alpha\beta$), along with a zero-sequence component. This transformation simplifies three-phase system analysis, by reducing the number of variables while maintaining the essential information.

The mathematical elegance and practical utility of these transformations have made them ubiquitous in power systems analysis. Modern applications range from Field-Oriented Control (FOC) of AC machines [1], Space Vector Pulse Width Modulation (SVPWM) for inverters [2], Direct Torque Control (DTC) strategies, grid-connected converter control under unbalanced conditions [3], active power filter design [4], power quality monitoring and assessment [5], fault analysis in power systems [6] to renewable energy system integration [7].

1.2 Educational Motivation and Pedagogical Need

Despite their widespread use across virtually every subdomain of power electronics and electric drives, educational materials often present them in one of two ways: either as matrix operations without a geometric foundation, or embedded within machine-specific derivations that obscure the general mathematical structure. While rigorous derivations exist in advanced textbooks [8], and recent tutorial papers [9] and [10], there remains significant pedagogical value in a unified treatment that:

- Begins from the first principles with general three-phase voltage equations in the time domain;
- Develops 3D spatial vector representation systematically, showing how balanced quantities reside naturally in a specific plane;
- Derives Clarke transformation through explicit geometric projection with clear visualization;
- Connects the Park transformation as a time-varying rotation of the Clarke frame;
- Explains the physical meaning of the key parameters, such as the 35.26° angle and transformation coefficients;
- Provides a worked example suitable for classroom instruction and self-study;
- Addresses practical considerations for digital implementation.

Students in power electronics and electric machines courses benefit from seeing the complete logical progression, from basic three-phase equations to transformation matrices, with explicit geometric interpretation at each step.

1.3 Pedagogical Contribution

This work provides multiple angles of pedagogical contribution for power electronics and electric machines education:

For the Educators: A complete, systematic derivation suitable for graduate-level courses in power electronics, electric machines, and power systems analysis; Geometric visualizations and 3D representations that enhance student intuition; Worked numerical examples ready for classroom use or homework assignments; Practical implementation guidance connecting theory to digital control systems.

For the Students: Clear logical progression from familiar three-phase equations to transformation matrices; Explicit geometric interpretation reducing reliance on memorization; Connection between mathematical operations and physical machine/system behavior.

1.4 Paper Organization

The remainder of this paper proceeds as follows: Section 2 establishes a mathematical framework by presenting general three-phase voltage representations in the time domain, developing a spatial vector form in a three-dimensional space. Section 3 provides a systematic derivation of the Clarke transformation. Section 4 provides a systematic derivation of the Park transformation, explains the frequency translation property, and presents a complete worked numerical example. Section 5 provides a conclusion with a summary of the pedagogical contributions.

2 Mathematical Framework

2.1 Three-Phase Voltage Representations

Three-phase voltages in general form and with arbitrary harmonic content can be expressed as in (1.1):

$$
\begin{aligned}
v_a(t) &= \sum_{h=0}^{\infty} V_{am,h} \cos(h\omega t - hc\frac{\pi}{6} + \varphi_{a,h}) \\
v_b(t) &= \sum_{h=0}^{\infty} V_{bm,h} \cos(h\omega t - hc\frac{\pi}{6} + \varphi_{b,h} - h\tfrac{2\pi}{3}) \\
v_c(t) &= \sum_{h=0}^{\infty} V_{cm,h} \cos(h\omega t - hc\frac{\pi}{6} + \varphi_{c,h} + h\tfrac{2\pi}{3})
\end{aligned}
\tag{1.1}
$$

where $V_{am,h}$, $V_{bm,h}$, $V_{cm,h}$ represent amplitudes of the h-th harmonic in each phase, $\omega = 2\pi f$ is the fundamental angular frequency, c is the clock number of the three-phase system, and $\varphi_{a,h}$, $\varphi_{b,h}$, $\varphi_{c,h}$ are the phase angles of the h-th harmonic.

A three-phase system can be represented in a three-dimensional plane, where each phase is associated with each dimension in the way presented in (1.2):

$$\begin{bmatrix} \overrightarrow{V_a(t)} \\ \overrightarrow{V_b(t)} \\ \overrightarrow{V_c(t)} \end{bmatrix} = \begin{bmatrix} \vec{V}_a & \vec{V}_b & \vec{V}_c \end{bmatrix} \begin{bmatrix} \sum_{h=0}^{\infty} V_{am,h} \cos(h\omega t - hc\frac{\pi}{6} + \varphi_{a,h}) \\ \sum_{h=0}^{\infty} V_{bm,h} \cos(h\omega t - hc\frac{\pi}{6} + \varphi_{b,h} - h\frac{2\pi}{3}) \\ \sum_{h=0}^{\infty} V_{cm,h} \cos(h\omega t - hc\frac{\pi}{6} + \varphi_{c,h} + h\frac{2\pi}{3}) \end{bmatrix} \tag{1.2}$$

where $\overrightarrow{V_a(t)}$, $\overrightarrow{V_b(t)}$, $\overrightarrow{V_c(t)}$ represents the resulting phasor of phases *a*, *b*, and *c* in a three-dimensional plane, and $\vec{V}_a$, $\vec{V}_b$, $\vec{V}_c$ represent the ort vectors for each dimension.

The time dependence of the resulting three-phase voltage vector in a three-dimensional plane is obtained by summing up the phasors. The amplitude invariant voltage vector is a three-dimensional plane obtained using (1.3) as:

$$\overrightarrow{V_g(t)} = \sqrt{\frac{2}{3}}(\overrightarrow{V_a(t)} + \overrightarrow{V_b(t)} + \overrightarrow{V_c(t)}) \tag{1.3}$$

Based on (1.2) and (1.3), the balanced harmonic in a three-phase system forms a voltage vector in a three-dimensional space whose trajectory obeys the following rules [11]:

1. Harmonics not divisible by 3 form a circle that lies in a plane (1.4)

$$\overrightarrow{V_a(t)} + \overrightarrow{V_b(t)} + \overrightarrow{V_c(t)} = 0 \tag{1.4}$$

1.1. $3h+1, h \in \overline{1,\infty}$ harmonics have one direction of rotation

1.2. $3h+2, h \in \overline{1,\infty}$ harmonics have the opposite direction of rotation

2. Harmonics divisible by 3 ($3h, h \in \overline{1,\infty}$) form a line that lies on a line perpendicular to the previous plane, and has the mathematical description presented in (1.5)

$$\overrightarrow{V_a(t)} = \overrightarrow{V_b(t)} = \overrightarrow{V_c(t)} \tag{1.5}$$

The relative geometry of positive vector orientation, plane (1.4) and line (1.5) is shown in Figures 1 a) and b). The positive vector orientation forms an angle of 35.26° with the plane (1.4). This angle arises naturally from the geometric constrain that three balanced phase vectors must have zero sum. In a three-dimensional *abc* space, consider three-unit vectors representing the phase axes: $\vec{V}_a = [1,0,0]$ (along the phase A axis), $\vec{V}_b = [0,1,0]$ (along the phase B axis), and $\vec{V}_c = [0,0,1]$ (along the phase C axis). A balanced plane is defined as $\overrightarrow{V_a(t)} + \overrightarrow{V_b(t)} + \overrightarrow{V_c(t)} = 0$. The normal vector to this plane is $\vec{N} = [1,1,1]/\sqrt{3}$. The angle θ between the phase A axis ($\vec{V}_a$) and the balance plane equals the complementary angle to that between $\vec{V}_a$ and the normal $\vec{N}$: $\cos(90° - \theta) = \vec{V}_a \cdot \vec{N} = [1,0,0] \cdot [1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}] = 1/\sqrt{3}$. Therefore, $\sin(\theta) = 1/\sqrt{3}$. Using the Pythagorean identity: $\cos(\theta) = \sqrt{2/3}$. The angle between the balanced plane and the phase A, B or C axes can be calculated as $\theta = arctan(1/\sqrt{2}) = 35.26°$. This angle is fundamental to understanding the transformation coefficients, which arise as projections at this specific angle.

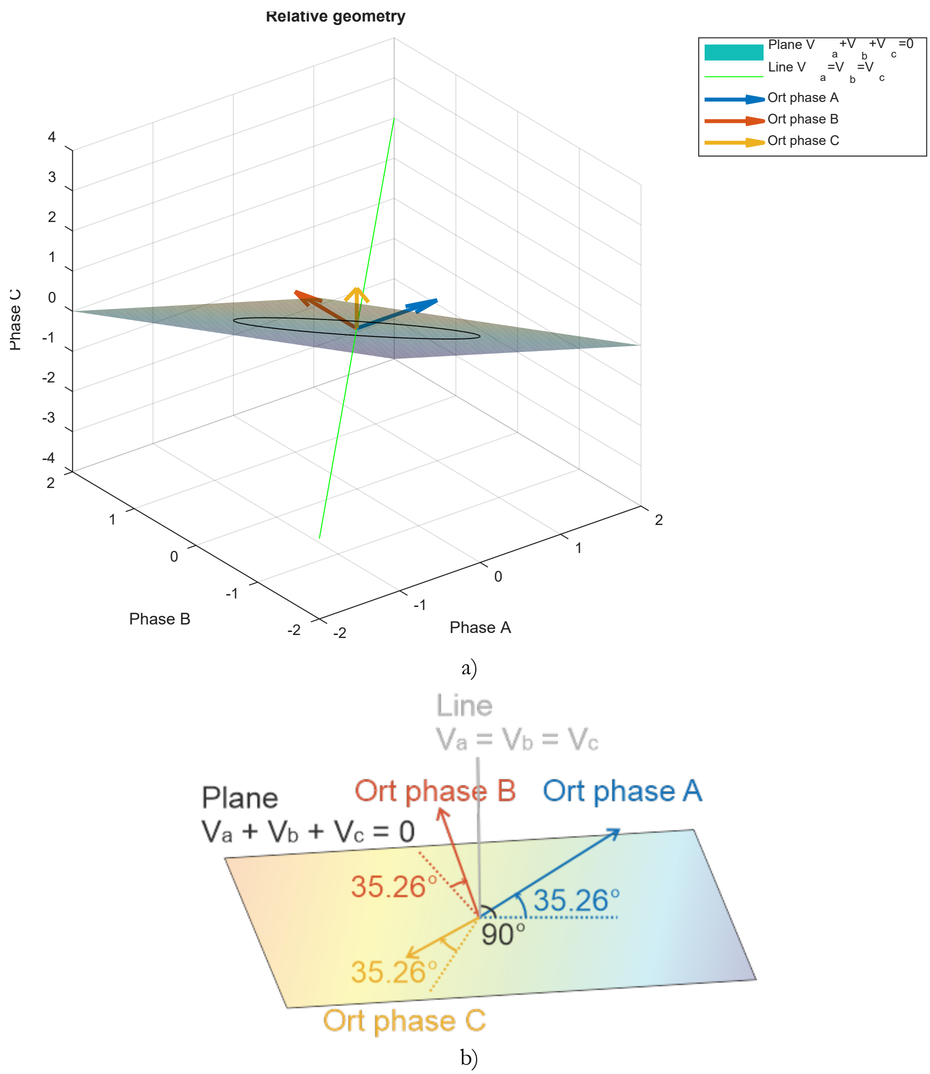


Figure 1: Three-Dimensional Geometry of Three-Phase Systems

Source: own.

3 Clarke Transformation

Based on the analysis from the previous section, it can be concluded that, in the three-phase space, the trajectories of the integer multiples of harmonics exist either in the plane defined by equation, (1.4) or on line (1.5). This observation motivates the introduction of a new coordinate system that simplifies the representation of three-phase voltage systems.

We adopt a three-axis coordinate system whose two independent axes lie in the plane (1.4), such that:

- The α-axis is collinear with the projection of the positive orientation of phase A onto the plane (1.4);
- The β-axis lies in the plane (1.4), leading the α-axis by 90°;
- The 0-axis is collinear with the zero-sequence line (1.5).

The graphical representation of this adopted coordinate system is shown in Figure 2. Based on Figure 1. b) the voltage projection of the *abc* system on the newly defined αβ0 system can be formulated as:

$$\vec{V}_\alpha(t) = \sqrt{\frac{2}{3}} \cdot (\vec{V}_a(t)\cdot\cos(0) + \vec{V}_b(t)\cdot\cos(-\frac{2\pi}{3}) + \vec{V}_c(t)\cdot\cos(\frac{2\pi}{3}))\cdot\cos(35.26^\circ)$$
$$\vec{V}_\beta(t) = \sqrt{\frac{2}{3}} \cdot (\vec{V}_a(t)\cdot\sin(0) + \vec{V}_b(t)\cdot\sin(\frac{2\pi}{3}) + \vec{V}_c(t)\cdot\sin(-\frac{2\pi}{3}))\cdot\cos(35.26^\circ)$$
$$\vec{V}_0(t) = \sqrt{\frac{2}{3}} \cdot (\vec{V}_a(t) + \vec{V}_b(t) + \vec{V}_c(t))\cdot\sin(35.26^\circ)$$

(1.6)

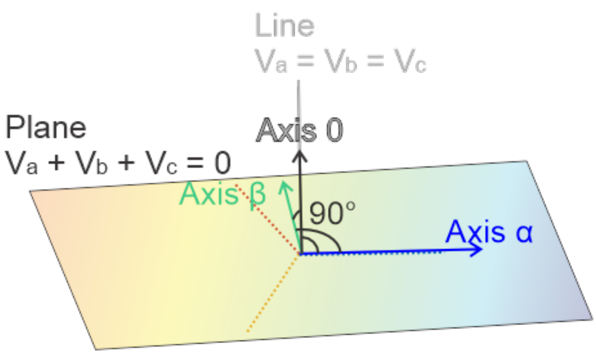


Figure 2: Clarke transformation reference frame (αβ0 system)

Source: own.

By substituting $\cos(35.26^\circ)=\sqrt{2/3} \wedge \sin(35.26^\circ)=\sqrt{1/3}$, expression (1.6) can be written in matrix form (1.7) with a new matrix defined in (1.8):

$$\begin{bmatrix} \vec{V}_\alpha(t) \\ \vec{V}_\beta(t) \\ \vec{V}_0(t) \end{bmatrix} = \frac{2}{3}\left[T_{\alpha\beta 0,abc}\right] \begin{bmatrix} \vec{V}_a(t) \\ \vec{V}_b(t) \\ \vec{V}_c(t) \end{bmatrix} \tag{1.7}$$

$$\left[T_{\alpha\beta 0,abc}\right] = \begin{bmatrix} \cos(0) & \cos(-\frac{2\pi}{3}) & \cos(\frac{2\pi}{3}) \\ \sin(0) & \sin(\frac{2\pi}{3}) & \sin(-\frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{1.8}$$

The previous expression represents an amplitude-invariant Clarke transformation. Coefficient $2/3$ in (1.7) is often denoted as a scaling factor *K* in the literature, and its variants, along with the area where the specific scaling factor finds application, is presented in Table 1 [11].

Table 1: Clarke Transformation Variants

	Scaling factor *K*	Property Preserved	Typical Applications
Amplitude-Invariant	$2/3$	Peak voltage/current magnitudes	Motor control (FOC), AC current/voltage control
Power-Invariant	$\sqrt{2/3}$	Instantaneous power	Power systems analysis, Power quality
Simplified	$\sqrt{1/3}$	Unit transformation (orthonormal)	Theoretical analysis, Symmetrical components

Source: own

4 Park Transformation

The Clarke transformation establishes a stationary reference frame (αβ0) where two axes lie in the plane of the balanced three-phase quantities. However, in this stationary frame, the balanced AC quantities still appear as sinusoidally varying signals. For many control applications–particularly field-oriented control of AC machines–it is advantageous to work with DC quantities rather than AC signals.

The previous transformation envisages fixed positioning of the coordinate system axes relative to the orientation of the three-phase system axes. The idea of forming a modified coordinate system can be considered, in which two axes lie in the same plane as αβ, but the orientation of the axes changes over time — the axes rotate with some angular velocity $\hat{\omega}$. Of the two axes that lie in the same plane as the αβ axes, we define the *d*-axis and the *q*-axis in the plane to lead by 90° relative to *the d-axis*. As in the case of the 0 αβ system, let the 0-axis remain, oriented perpendicular to the newly formed *dq* plane. A graphical illustration of the coordinate system is shown in Figure 3.

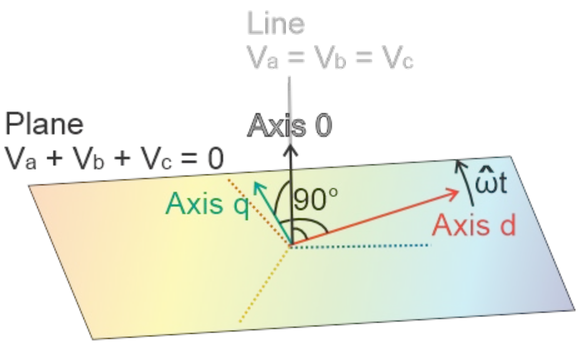


Figure 3: Park transformation reference frame (*dq*0 system)
Source: own.

In the same way as for the Clarke transformation, the voltage projection on the newly defined *d* and *q* axes can be formulated in matrix form as in (1.9) and (1.10):

$$\begin{bmatrix} \vec{V}_d(t) \\ \vec{V}_q(t) \\ \vec{V}_0(t) \end{bmatrix} = \frac{2}{3}\left[T_{dq0,abc}\right]\begin{bmatrix} \vec{V}_a(t) \\ \vec{V}_b(t) \\ \vec{V}_c(t) \end{bmatrix} \tag{1.9}$$

$$\left[T_{dq0,abc}\right] = \begin{bmatrix} \cos(\hat{\omega} t) & \cos(\hat{\omega} t - \frac{2\pi}{3}) & \cos(\hat{\omega} t + \frac{2\pi}{3}) \\ -\sin(\hat{\omega} t) & -\sin(\hat{\omega} t - \frac{2\pi}{3}) & -\sin(\hat{\omega} t + \frac{2\pi}{3}) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \tag{1.10}$$

Or, in the case when the transformation to the *dq*0 system is done from the αβ0 system, the transformation matrix has the form presented in (1.11) and (1.12):

$$\begin{bmatrix} \vec{V}_d(t) \\ \vec{V}_q(t) \\ \vec{V}_0(t) \end{bmatrix} = \begin{bmatrix} T_{dq0,\alpha\beta0} \end{bmatrix} \begin{bmatrix} \vec{V}_\alpha(t) \\ \vec{V}_\beta(t) \\ \vec{V}_0(t) \end{bmatrix} \tag{1.11}$$

$$\begin{bmatrix} T_{dq0,\alpha\beta0} \end{bmatrix} = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) & 0 \\ -\sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1.12}$$

The expressions (1.9) and (1.10), or (1.11) and (1.12) represent an amplitude-invariant Park transformation. Different values of the scaling factor *K* presented in Table 1 can be applied as in the case of the Clarke transformation. Physical interpretation of the Park transformation:

- When the *dq* frame rotates at the same frequency as the αβ voltage/current vectors (synchronous rotation), the projections onto *d* and *q* axes become constant (DC);
- The *d*-axis is, typically, aligned with a meaningful reference (e.g., the rotor flux in FOC);
- The *q*-axis represents the component in quadrature (90°) to the reference.

The Park transformation also acts as a frequency translator:

- The fundamental frequency (ω) values are seen as DC in the synchronous rotating frame;
- DC can be seen as a negative fundamental ($-\omega$);
- The 5th harmonic (5ω) in the *abc* frame as the 6th harmonic with the opposite direction of rotation (-6ω) in the *dq* frame (odd harmonics have an opposite direction of rotation in the plane relative to the fundamental harmonic);

- The 7th harmonic (7ω) in the *abc* frame as the 6th harmonic (6ω) in the *dq* frame (even harmonics have the same direction of rotation in the plane relative to the fundamental harmonic);

To solidify understanding, we present a complete worked example demonstrating the transformation of a balanced three-phase system through both the Clarke and Park transformations.

Given System: A balanced three-phase voltage system with fundamental harmonic only:

- RMS voltage: 100 V per phase
- Peak voltage: $V_m = 100\sqrt{2} = 141.42$ V
- Frequency: $f = 50$ Hz
- Angular frequency: $\omega = 2\pi f = 314.16$ rad/s
- Initial phase of the grid voltage vector (at $t = 0$): $\omega t - c\frac{\pi}{6} + \varphi_a = 0$
- Initial Phase of the Park transformation reference frame (at $t = 0$): $\omega t = 0$ (*d*-axis aligned with phase A)

Calculation of grid voltages in the *abc* domain at $t = 0$ using (1.1) is presented in (1.13):

$$
\begin{aligned}
v_a(0) &= 141.42\cos(0) &&= 141.42 \\
v_b(0) &= 141.42\cos(0 - 120°) &&= -70.71 \\
v_c(0) &= 141.42\cos(0 + 120°) &&= 70.71
\end{aligned}
\tag{1.13}
$$

When the amplitude-invariant Clarke transformation, presented in (1.7), is applied to the instantaneous values obtained in (1.13), the following αβ0 values can be obtained, as in (1.14):

$$\begin{bmatrix} \vec{V}_\alpha(0) \\ \vec{V}_\beta(0) \\ \vec{V}_0(0) \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(0) & \cos(-\frac{2\pi}{3}) & \cos(\frac{2\pi}{3}) \\ \sin(0) & \sin(\frac{2\pi}{3}) & \sin(-\frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 141.42 \\ -70.71 \\ 70.71 \end{bmatrix} = \begin{bmatrix} 141.42 \\ 0 \\ 0 \end{bmatrix} \tag{1.14}$$

When the amplitude-invariant Park transformation from (1.9) is applied to(1.13), (1.15) can be obtained:

$$\begin{bmatrix} \vec{V}_d(t) \\ \vec{V}_q(t) \\ \vec{V}_0(t) \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(0) & \cos(0-\frac{2\pi}{3}) & \cos(0+\frac{2\pi}{3}) \\ -\sin(0) & -\sin(0-\frac{2\pi}{3}) & -\sin(0+\frac{2\pi}{3}) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 141.42 \\ -70.71 \\ 70.71 \end{bmatrix} = \begin{bmatrix} 141.42 \\ 0 \\ 0 \end{bmatrix} \tag{1.15}$$

5 Conclusion

This paper has presented the comprehensive mathematical derivations of the Clarke and Park transformations, from first principles through systematic vector representation. By progressing from three-phase voltage equations to spatial vectors in a three-dimensional *abc* space, we established the rigorous geometric foundations underlying these ubiquitous transformations.

The Clarke transformation emerges as an orthogonal projection from the three-dimensional *abc* space onto the plane $\overrightarrow{V_a(t)} + \overrightarrow{V_b(t)} + \overrightarrow{V_c(t)} = 0$ where balanced quantities reside. The characteristic 35.26° angle and transformation coefficients derive directly from geometric principles, with the amplitude-invariant ($\sqrt{2/3}$ scaling) formulation derived rigorously. The Park transformation follows as a time-varying rotation of the Clarke frame, creating a frequency translation that converts the AC quantities to DC under synchronous rotation.

Coordinate transformations are not merely mathematical tools, but represent fundamental insights about the geometry of three-phase systems. By understanding these transformations from the first principles through systematic geometric derivation, the students and engineers develop a deeper intuition, that enhances their ability to work with modern power electronic and motor control systems.

Acknowledgment

This work has been supported by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia [Grant Number:451-03-136/2025-03/200102]

References

[1] D. W. Novotny and T. A. Lipo, *Vector Control and Dynamics of AC Drives*. Oxford University PressOxford, 1996. doi: 10.1093/oso/9780198564393.001.0001.

[2] F. Blaabjerg, R. Teodorescu, M. Liserre, and A. V. Timbus, "Overview of Control and Grid Synchronization for Distributed Power Generation Systems," *IEEE Transactions on Industrial Electronics*, vol. 53, no. 5, pp. 1398–1409, Oct. 2006, doi: 10.1109/TIE.2006.881997.

[3] X. Wang, F. Blaabjerg, and W. Wu, "Modeling and Analysis of Harmonic Stability in an AC Power-Electronics-Based Power System," *IEEE Trans Power Electron*, vol. 29, no. 12, pp. 6421–6432, Dec. 2014, doi: 10.1109/TPEL.2014.2306432.

[4] H. Akagi, "Active Harmonic Filters," *Proceedings of the IEEE*, vol. 93, no. 12, pp. 2128–2141, Dec. 2005, doi: 10.1109/JPROC.2005.859603.

[5] A. Testa *et al.*, "Interharmonics: Theory and Modeling," *IEEE Transactions on Power Delivery*, vol. 22, no. 4, pp. 2335–2348, Oct. 2007, doi: 10.1109/TPWRD.2007.905505.

[6] D. Bellan, "Clarke Transformation Solution of Asymmetrical Transients in Three-Phase Circuits," *Energies (Basel)*, vol. 13, no. 19, p. 5231, Oct. 2020, doi: 10.3390/en13195231.

[7] M. Liserre, T. Sauter, and J. Hung, "Future Energy Systems: Integrating Renewable Energy Sources into the Smart Power Grid Through Industrial Electronics," *IEEE Industrial Electronics Magazine*, vol. 4, no. 1, pp. 18–37, Mar. 2010, doi: 10.1109/MIE.2010.935861.

[8] P. Krause, O. Wasynczuk, S. Sudhoff, and S. Pekarek, Eds., *Analysis of Electric Machinery and Drive Systems*. Wiley, 2013. doi: 10.1002/9781118524336.

[9] F. G. Montoya and A. H. Eid, "Formulating the geometric foundation of Clarke, Park, and FBD transformations by means of Clifford's geometric algebra," *Math Methods Appl Sci*, vol. 45, no. 8, pp. 4252–4277, May 2022, doi: 10.1002/mma.8038.

[10] C. J. O'Rourke, M. M. Qasim, M. R. Overlin, and J. L. Kirtley, "A Geometric Interpretation of Reference Frames and Transformations: dq0, Clarke, and Park," *IEEE Transactions on Energy Conversion*, vol. 34, no. 4, pp. 2070–2083, Dec. 2019, doi: 10.1109/TEC.2019.2941175.

[11] F. Filipović, "Advanced Synchronization Algorithms for the Operation Improvement of Renewable Energy Source Grid Inverters," Faculty of Electronic Engineering, Niš, 2025.

About the authors

Filip Filipović, Aleksandra Medvedeva 4, Department of Electrical Power Engineering, Faculty of Electronic Engineering, University of Niš
Teaching Assistant

Milutin Petronijević, Aleksandra Medvedeva 4, Department of Electrical Power Engineering, Faculty of Electronic Engineering, University of Niš
Associate Professor

Nebojša Mitrović, Aleksandra Medvedeva 4, Department of Electrical Power Engineering, Faculty of Electronic Engineering, University of Niš
Full Professor

Bojan Banković, Aleksandra Medvedeva 4, Department of Electrical Power Engineering, Faculty of Electronic Engineering, University of Niš
Teaching Assistant

Anđela Stojiljković, Aleksandra Medvedeva 4, Department of Electrical Power Engineering, Faculty of Electronic Engineering, University of Niš
Researcher

Vojkan Kostić, Aleksandra Medvedeva 4, Department of Electrical Power Engineering, Faculty of Electronic Engineering, University of Niš
Assistant Professor

Povzetek v slovenskem jeziku

Transformacije z vektorsko predstavitvijo v trifaznih sistemih. Članek predstavlja celovito obravnavo Clarkove in Parkove transformacije skozi sistematično vektorsko predstavitev, zasnovano posebej za izobraževalne namene na področju močnostne elektronike in vodenja električnih strojev. Čeprav sta ti temeljni transformaciji vseprisotni v sodobnih aplikacijah – od poljsko usmerjenega vodenja AC strojev do vodenja omrežno priključenih pretvornikov – učni materiali pogosto predstavljajo transformacije bodisi kot matrične operacije brez geometrijske osnove, bodisi v okviru izpeljav, vezanih na določene tipe strojev, kar zamegli splošno matematično strukturo. V tem delu se posvetimo tej pedagoški vrzeli z doslednim prehodom od trifaznih napetostnih enačb v časovni domeni do prostorske vektorske predstavitve v tridimenzionalnem prostoru. Clarkovo transformacijo izpeljemo kot eksplicitno geometrijsko projekcijo na ravnino, v kateri ležijo uravnotežene veličine, nato pa Parkovo transformacijo kot časovno spremenljivo rotacijo Clarkovega koordinatnega sistema. Delo vzpostavi amplitudno-invariantno formulacijo, poda jasno geometrijsko razlago 35,26° kota med koordinatnimi sistemi ter vključuje numerične primere z vsemi koraki transformacij. Ta enotna pedagoška obravnava zapolnjuje vrzel med praktično uporabo in matematičnimi temelji ter ponuja pedagoškemu kadru popolnoma geometrijsko intuitiven okvir, primeren za podiplomsko poučevanje in strokovno usposabljanje.

JOURNAL OF ENERGY TECHNOLOGY
Vol. 18, No. 3, pp. 161–168, November 2025

Professional Article

RADIOACTIVE WASTE MANAGEMENT STRATEGY FOR A NEW NUCLEAR POWER PLANT

Submitted
24. 11. 2025

Accepted
26. 11. 2025

Published
28. 11. 2025

EVA BAHČIČ, KAJA NAGLIČ, TOMAŽ ŽAGAR
GEN energija d.o.o, Krško, Slovenia
eva.bahcic@gen-energija.si, kaja.naglic@gen-energija.si, tomaz.zagar@gen-energija.si

CORRESPONDING AUTHOR
eva.bahcic@gen-energija.si

Abstract An essential early phase of any new nuclear power plant (NPP) project is the drafting and preparation of a radioactive waste management strategy. The paper outlines the major steps in such a strategy, presented in a case study approach for the new NPP project in Krško (the JEK2 project). The paper includes analyses of the options and strategies for radioactive waste (RAW) and spent nuclear fuel (SNF) management. This article presents a proposed management plan for the RAW and SNF expected to be produced during the operation and decommissioning of JEK2, together with an assessment of the anticipated decommissioning and waste management costs. The document examines further the existing financing framework for radioactive waste management in Slovenia, which is regulated legally for the current nuclear facilities. A similar approach is planned for JEK2, including the expansion of the existing and planned facilities for waste handling and storage. Accordingly, additional financial resources will be required through contributions to the national decommissioning and disposal fund. An analysis was therefore conducted, to estimate the scale of the additional contributions needed to ensure adequate funding for the new NPP project in Krško.

Keywords
JEK2,
radioactive waste,
spent nuclear fuel,
storage,
fund

https://doi.org/10.18690/jet.18.3.161-168.2025

University of Maribor Press

1 Management of Radioactive Waste and Spent Nuclear Fuel and Assessment of the Fund Contributions

RAW is generated at any nuclear power plant (NPP) from the beginning of the trial operation, while SNF is first generated only after the completion of the plant's initial fuel cycle and the subsequent refuelling outage. The management of RAW and SNF from the new NPP project in Krško (JEK2) will be planned in accordance with the principles and guidelines of Resolution ReNPROIG23–32, or in accordance with the requirements of the resolution in force at the time the application is submitted [1], [2].

In Slovenia, waste management programmes will be prepared in accordance with the Regulation on Radiological and Nuclear Safety Factors, and will comply with the content requirements of the Regulation on the Management of Radioactive Waste and Spent Fuel [1], [2].

1.1 Storage and disposal of low- and intermediate-level radioactive waste

Based on their origin and characteristics, the following types of operational low- and intermediate-level radioactive waste (LILW) can be expected for an NPP: waste concentrates and sludges from the liquid systems, spent ion-exchange resins, used filter cartridges, compressible waste, and various solid and non-compressible waste. The waste concentrates, sludges and resins will be solidified prior to storage, either through drying, or by treatment with binding materials [1], [6], [7].

The LILW generated at JEK2 will, in accordance with the planned schedule, be transferred to the Agency for Radioactive Waste Management (ARAO) for disposal. The ARAO will operate the LILW Repository. The LILW Repository will be designed as a near-surface facility in the form of an underground silo, which will be filled from the surface. The standard storage packaging for LILW will consist of steel drums of various designs, made of either carbon or stainless steel. Prior to disposal, all the LILW will be placed into disposal containers, such as the N2d-type containers [1], [6], [7].

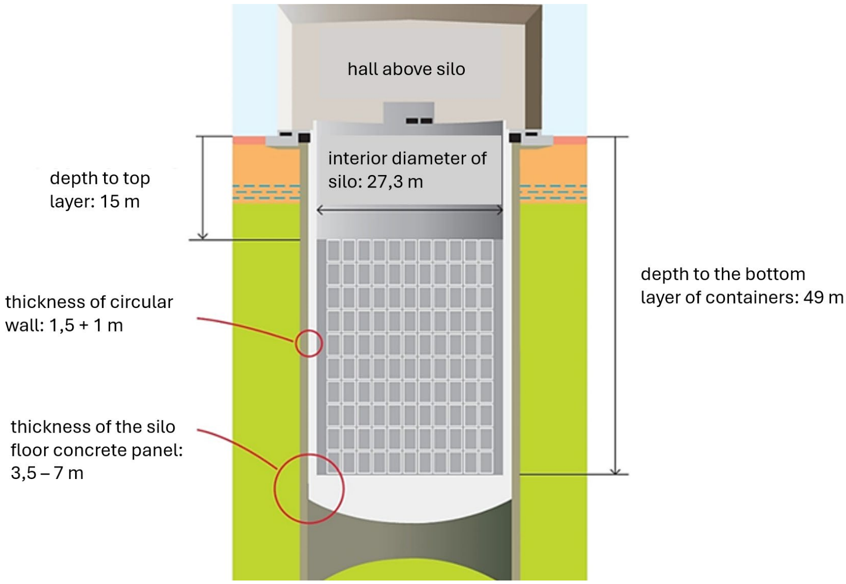


Figure 1: LILW disposal facilities in the form of a near-surface silo with an above-ground hall.

The first silo will have the capacity to accommodate 990 N2d-type containers, arranged in ten disposal layers. The interstitial spaces between the containers will be filled progressively with backfill material, and every second layer will be covered with a concrete levelling layer. The uppermost layer will have a thickness of one metre. Upon closure of the repository, the space above the silo will be filled almost to the surface with clay, which will serve to prevent water infiltration from above. During the operational phase, the silo will remain open at the top and protected by a surface enclosure. All above-ground structures will be dismantled following the closure of the repository [1], [7].

The management of LILW within the JEK2 project in Slovenia will, from a technological standpoint, represent an upgrade of the currently planned approach described in the Resolution on the National Programme for the Management of Radioactive Waste and Spent Fuel (ReNPROIG23-32) [2].

During its operational phase, the JEK2 project will necessitate an increase in disposal capacity, requiring the acquisition of appropriate construction permits for the existing disposal site. This will involve expanding the capacity of the current repository rather than constructing a new facility. Nevertheless, the remaining available capacity of the first silo will suffice for at least several years of operational LILW disposal from JEK2. In any case, with the addition of one extra silo for JEK2—already designated spatially —there will be sufficient capacity to cover 30 years of JEK2 operation [1], [7] .

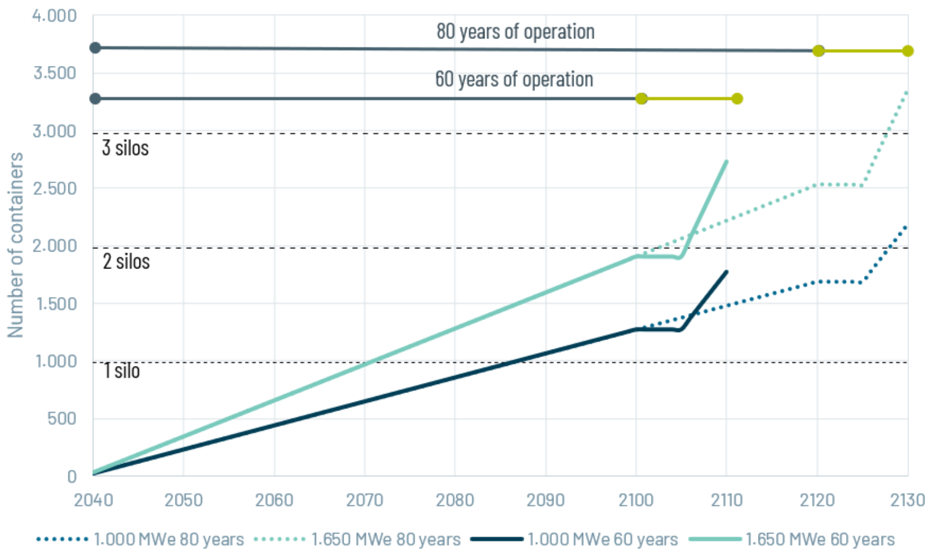


Figure 2: Display of the storage capacities for LILW.

The planned expansion will include the construction of additional silos, and will entail a corresponding increase in the repository's operational costs.

Assuming identical disposal conditions to those applied for the first LILW disposal silo, the management of the LILW generated by the new NPP project in Krško (JEK2), with a unit capacity of 1.000 Mwe, would necessitate the construction of two additional silos. For units with capacities of 1.250 MWe and 1.650 MWe, three additional silos would be required to accommodate the waste generated during a 60-year operational lifetime.

Table 1: LILW quantities and conservative estimate of silos [1], [7].

	JEK2 power 1.000 MWe	JEK2 power 1.250 MWe	JEK2 power 1.650 MWe
operational LILW (m^3)	3.000	3.577	4.500
LILW from decommissioning (tones)	5.000	6.250	8.250
Number of N2d containers for operational LILW	1.250	1.490	1.875
Number of N2d containers for LILW from decommissioning	500	625	825
Total number of N2d containers	1.750	2.115	2.700
Number of additional silos required (conservative estimate)	**1,77**	**2,14**	**2,73**

1.2 Dry storage and disposal of high-level radioactive waste

The SNF is first stored in a pool next to the reactor. The pool is designed to store sufficient capacity for at least 15 years of operation, considering an appropriate reserve, and space is always available in the pool for the insertion of the entire reactor core. The spent fuel will be stored in a water-cooled pool, and the heat will be dissipated into the environment by an active cooling system.

The JEK2 project, presented in this paper as an example of a new nuclear power plant construction project currently under discussion in Slovenia, also includes a facility for the dry storage of the SNF generated during the entire planned lifetime of the power plant. The SNF can be transferred to dry storage at least five years after removal from the reactor and cooling in the spent fuel pool. For the purposes of dry storage, the fuel elements will be removed from the pool and placed in multi-purpose containers in the building next to the reactor. The containers with SNF will be transferred to the dry storage building and placed in storage casks. These will provide passive cooling of the SNF with ambient air [1], [2].

The dry storage facility will be used to store the SNF until a deep geological repository is established, ensuring adequate temporal isolation of the waste from the environment.

Table 2: Number of fuel elements required for 60 years of operation.

	JEK2 power 1.000 MWe	JEK2 power 1.250 MWe	JEK2 power 1.650 MWe
Number of fuel elements – 60 years of operation	3.348	4.646	4.831

The fuel elements will be inserted into channels inside special containers. For fuel elements in PWR reactors, such as the planned JEK2, the design is such that each container has four separate channels, allowing four fuel elements to be placed in a single container. Based on the estimated number of fuel elements, we calculated the number of containers required for a power range from 1.000 MW to 1.650 MW.

Table 3: Number of required containers for 60 years of operation [1].

	JEK2 power 1.000 MWe	JEK2 power 1.250 MWe	JEK2 power 1.650 MWe
Number of required containers – 60-year operating life	837	1.161	1.208

1.3 Economic evaluation

The total cost of decommissioning for the JEK2 project, as an example of new nuclear power plant construction project in Slovenia, and additional funds for the LILW and HLW is currently estimated at an additional 1.386–2.250 million EUR for a 60-year operating period. Considering the 1.5% real target return on the existing fund (3.0% nominal with an expected inflation rate of 1.5%), an estimated 14.4 – 23.4 EUR million would need to be collected annually for this purpose, which means 1.62–1.83 EUR per MWh of electricity produced over a 60-year operating period. However, due to the high degree of uncertainty surrounding decommissioning and disposal, a significantly higher value was considered conservatively in the economic model, namely, a decommissioning compensation of 2.0 EUR/MWh, due to the uncertainties regarding profitability and the final costs of waste disposal and decommissioning [1], [4], [7].

Table 4: Estimated costs for radioactive waste management for 60 years of operation.

	Additional funds[1] 1.000 MWe	Additional funds 1.250 MWe	Additional funds 1.650 MWe
	[million EUR]	[million EUR]	[v million EUR]
LILW	236	305	318
HLW	330 (clay)	402 (clay)	474 (clay)
	402 (hard geological formations)	502 (hard geological formations)	578 (hard geological formations)
DECOMISSIONING	820	1.026	1.354
TOTAL	**1.386** (clay)	**1.733** (clay)	**2.146** (clay)
	1.458 (hard geological formations)	**1.833** (hard geological formations)	**2.250** (hard geological formations)

Table 5: Contribution to the fund for 60 years of operation [1].

	1.000 MWe	1.250 MWe	1.650 MWe
Contribution to the fund [EUR/MWh] (clay)	1,73	1,73	1,62
Contribution to the fund [EUR/MWh] (hard geological formations)	1,82	1,83	1,70
Estimated evaluation of the contribution to the fund [EUR/MWh]	**2,00**		

References

[1][1] IBE d.d. and Sava project Krško d.d., Proposal for a national spatial plan for JEK2, 2025

[2][2] Resolution on the National Program for Radioactive Waste and Spent Fuel Management for the Period 2023-2032 (ReNPROIG23-32), Resolucija o nacionalnem programu ravnanja z radioaktivnimi odpadki in izrabljenim gorivom za obdobje 2023–2032 (ReNPROIG23–32) (PISRS)

[3][3] ARAO, Third Revision of the Krško NPP Radioactive Waste and Spent Fuel Disposal Program, version 1.3, September 2019

[1] Additional funds needed for the new NPP

[4] Rebekka Bärenbold et al. 2024, Renewable and Sustainable Energy Reviews, Decommissioning of commercial nuclear power plants: Insights from a multiple-case study
[5] Regulations on the management of radioactive waste and spent fuel, No. 125/21 and 74/24, Pravilnik o ravnanju z radioaktivnimi odpadki in izrabljenim gorivom (PISRS)
[6] ARAO, Safety report for the LILW Vrbina repository, 2021
[7] Internal source, GEN energije d.o.o., https://jek2.si/ravnanje-z-radioaktivnimi-odpadki/

Povzetek v slovenskem jeziku

Predvideno ravnanje z radioaktivnimi odpadki in izrabljenim gorivom iz JEK2 ter s tem povezanimi stroški. Ključen del priprav projekta JEK2 predstavljajo analize, potrebne za dolgoročno, varno in učinkovito načrtovanje ravnanja z radioaktivnimi odpadki (RAO), izrabljenim gorivom (IG) in razgradnjo elektrarne po koncu obratovanja. Članek na primeru projekta JEK2 predstavi glavne elemente takšne strategije. Vključuje pregled možnosti in pristopov k ravnanju z RAO in IG, za katere se pričakuje, da bodo nastali med obratovanjem in razgradnjo JEK2, skupaj z oceno predvidenih stroškov. Dokument dodatno preučuje obstoječi finančni okvir za ravnanje z radioaktivnimi odpadki v Sloveniji, ki je zakonsko urejen za obstoječe jedrske objekte. Podoben pristop je predviden za JEK2, vključno s širitvijo obstoječih in načrtovanih objektov za ravnanje z odpadki, njihovo skladiščenje in odlaganje. Ekonomski vidik financiranja ravnanja z RAO temelji na uveljavljeni praksi zbiranja sredstev prek namenskega sklada, ki deluje kot mehanizem za upravljanje finančnih virov.

JOURNAL OF ENERGY TECHNOLOGY
Vol. 18, No. 3, pp. 169–176, November 2025

Professional Article

DESIGN AND FABRICATION OF A DEMONSTRATION MODEL FOR PROMOTING LOW-CARBON ENERGY TECHNOLOGIES

MARKO PEZDEVŠEK,[1] GARSIA KOSINAC,[2]
BOŠTJAN PIŠOTEK,[3] ANDREJ PREDIN,[1] ANDRAŽ ROGER,[1]
MATEJ FIKE,[1] GORAZD HREN,[1]

[1] University of Maribor, Faculty of Energy Technology, Krško, Slovenia
marko.pezdevsek@um.si, andrej.predin@um.si, andraz.roger@um.si,
matej.fike@um.si, gorazd.hren@um.si
[2] GEN energija, Krško, Slovenia
garsia.kosinac@gen-energija.si
[3] HESS d.o.o., Brežice, Slovenia
bostjan.pisotek@he-ss.si

CORRESPONDING AUTHOR
marko.pezdevsek@um.si

Submitted
1. 10. 2025

Accepted
26. 10. 2025

Published
28. 11. 2025

Abstract This article presents the design, construction and educational application of a large-scale model representing renewable energy facilities that do not yet exist. The model features a circular layout with a diameter of 2 metres, divided into two halves: the Mokrice Hydropower Plant and a nuclear power plant with an accompanying low- and intermediate-level radioactive waste repository. The Mokrice half highlights key ecological and technical features, including two fish passages and a solar power plant installation, with realistic water effects achieved using epoxy resin. The nuclear half includes a removable low- and intermediate-level radioactive waste repository, allowing viewers to explore the storage arrangements via a cross-sectional view. All the components were fabricated using FDM 3D printing, assembled, finished with paint and landscaping materials, and mounted on a wooden base with a metal support structure. While the model was not intended to achieve exact technical scaling, it communicates complex energy infrastructures effectively, facilitating public understanding, awareness and dialogue about low-carbon technologies. The combination of additive manufacturing, interactive features and detailed landscape representation, demonstrates the value of models as tools for education, demonstration, and the promotion of sustainable energy solutions.

Keywords
model,
nuclear power plant,
hydro power plant,
3D printing,
CAD

https://doi.org/10.18690/jet.18.3.169-176.2025

University of Maribor Press

1 Introducton

Physical models are employed frequently as effective instruments for communication, education and public outreach. Their primary role is not necessarily to achieve technical precision or exact dimensional accuracy, but rather to provide a tangible medium through which complex infrastructures and technologies can be presented more comprehensibly to non-expert audiences. In this sense, the models serve as cognitive bridges, translating abstract or large-scale systems into accessible and visually engaging forms.

The demonstration model was commissioned by GEN energija, for promotional purposes, in collaboration with HESS and IBE, who are preparing the documentation for the Mokrice Hydropower Plant. The infrastructure of the Mokrice Hydropower Plant has been documented, while the nuclear aspect is still in the spatial planning phase. The model's design emerged from a collaborative brainstorming process involving the entire team of authors, balancing the client's requirements, technological constraints and the need for simplification in scaling.

A demonstration model was developed with the specific purpose of promoting the concept of a new nuclear power plant and illustrating its integration with associated infrastructure, such as a repository for low- and intermediate-level radioactive waste. Complementing this, the model also included a representation of the Mokrice Hydropower Plant, thereby demonstrating the coexistence of different energy technologies within a unified energy landscape. The aim of this model was not technical perfection or full-scale accuracy, but rather to raise public awareness, enhance understanding, and stimulate dialogue regarding nuclear energy and its role in low-carbon energy production.

2 Objective and purpose of the model

The primary goal of the model was to function as a communicative and educational tool rather than a technical prototype. Energy infrastructure models are often complex and abstract, making it difficult for non-specialist audiences to understand their structure, operation and interconnections fully. By transforming these systems into a tangible representation, the model helps the public to grasp the scale, spatial arrangement and functional principles of renewable and nuclear energy facilities.

The circular design, with a 2-metre diameter, provided a compact yet comprehensive display, dividing the model into two halves: the Mokrice Hydropower Plant and a nuclear power plant facility. This configuration was chosen deliberately, to allow simultaneous comparison and highlight the coexistence of diverse low-carbon energy technologies within a single energy landscape. The model's layout enables viewers to observe the spatial relationships, infrastructure integration and environmental context, fostering a more intuitive understanding than diagrams or photographs alone.

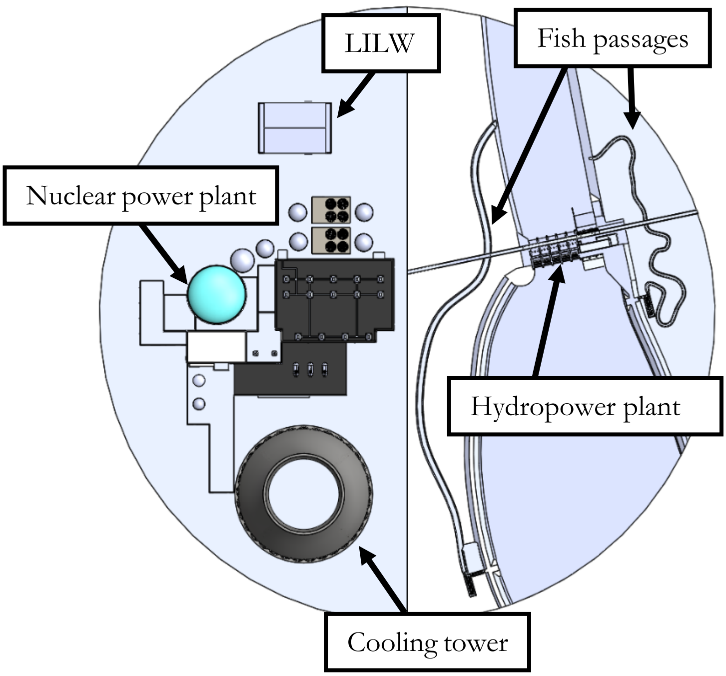


Figure 1: Model layout.

Moreover, the model emphasises the role of public outreach in energy planning. Engaging the public with accessible visualisations promotes transparency, encourages dialogue, and raises awareness about sustainable energy solutions. It also serves as a tool for educators, policymakers and industry stakeholders to communicate complex concepts effectively, thereby bridging the gap between technical knowledge and societal understanding.

3 Mokrice hydropower plant model

The Mokrice Hydropower Plant model is characterised by distinctive technical and ecological features. The facility incorporates two fish passages, designed to allow the safe migration of aquatic species and to maintain the river ecosystem connectivity, as well as a solar power plant installation integrated on the left side of the riverbank. These elements illustrate how renewable energy systems can combine hydropower with solar technology while addressing ecological considerations.

The original model was provided by IBE and adapted for 3D printing, requiring simplification of the geometric details to fit the circular layout. The adjustments included shortening certain distances, such as the left fish passage, without compromising the educational value of the representation. This process highlights the need to balance the physical constraints of model construction with fidelity to real-world characteristics. The printing was carried out on FDM (Fused Deposition Modelling) Prusa MK4 3D printers. Due to the printer size limitations, the model was divided into multiple parts, which were, later, assembled with adhesive and filler to ensure continuity and stability.

Figure 2: Combined parts that were filled with filler and sanded.

Post-processing included painting and texturing. Sand was applied to the riverbed, the stones along the embankments and static grass for vegetated areas. To represent the river, a layer of epoxy resin was applied to simulate water, giving the model a realistic appearance with depth and gloss. Trees and shrubs were added to create a realistic landscape, enhancing the model's visual and educational impact.

The completed hydropower half allows observers to examine the key functional components, spatial relationships and environmental integration. It demonstrates how a renewable energy infrastructure can coexist with natural ecosystems, serving as a model for public understanding of sustainable energy practices.

4 Nuclear power plant model

The nuclear half of the model provides a generalised representation of a nuclear energy facility, including a reactor, cooling tower, and a repository for low- and intermediate-level radioactive waste (LILW). A specific reactor type was not selected, reflecting the current uncertainty regarding the final contractor and reactor design.

A key educational feature is the removable LILW repository, which exposes a cross-section illustrating the arrangement of the storage casks. This feature enables the viewers to understand the spatial and safety considerations involved in nuclear waste management. It provides a rare opportunity for the public to visualise normally hidden or restricted facilities, making complex safety concepts more comprehensible.

Like the hydropower half, all the nuclear components were 3D printed, assembled, filled and painted. Roads, parking areas and ancillary infrastructure were added to provide context, while landscape features such as grass, trees and shrubs improved the realism. The nuclear half complements the renewable energy half, demonstrating the diversity of low-carbon technologies and highlighting the role of nuclear power in a balanced energy mix.

Figure 3: 3D printing of the cooling tower segments.

5 Assembly and final touches

The base structure was supplemented with a metal support fabricated by TEB - Termo Elektrarna Brestanica, providing stability for handling and display. The final model represents a compromise between scale, visual fidelity and practical construction constraints. Detailed finishing, including painting, static grass, stones and vegetation, allowed the model to convey realistic textures and environmental integration.

This assembly process demonstrates the interdisciplinary collaboration required for educational model creation, combining engineering, design, 3D printing technology and landscape modelling. The result is a visually compelling tool that communicates complex energy concepts effectively to diverse audiences.

Figure 4: Finished model.

6 Discussion and implications for low-carbon energy awareness

The completed model illustrates the complementary roles of hydropower, solar and nuclear technologies in achieving low-carbon energy goals. By presenting renewable and nuclear facilities together, the model highlights how different technologies can coexist within a unified energy strategy.

Educationally, the model provides an accessible entry point for the public to engage with a complex energy infrastructure. Interactive features, such as the removable waste repository, enable the viewers to explore normally inaccessible aspects, promoting understanding of the technical, safety and environmental considerations. The realistic depiction of landscape integration and technical features fosters a deeper appreciation for the planning, design and operation of energy facilities. It is important to emphasise the extensive work involved in preparing, detailing and setting up both the geometry and technological aspects for 3D printing.

Models of this type are valuable for public outreach, education and stakeholder engagement. They facilitate informed discussions, promote transparency and support communication, helping bridge the gap between technical knowledge and societal understanding. By combining additive manufacturing, detailed surface finishing and interactive elements, this model serves as an effective medium for demonstrating low-carbon technologies and raising awareness about sustainable energy solutions.

Acknowledgment

The authors would like to acknowledge the financial support provided by the GEN and HESS foundations (Grant No. GEN/23/2024), which made this work possible.

Povzetek v slovenskem jeziku

Zasnova in izdelava makete za promocijo nizkoogljičnih virov. Članek predstavlja zasnovo, izdelavo in izobraževalno uporabo makete, ki prikazuje obnovljive vire energije. Maketa ima krožno postavitev s premerom 2 metra, razdeljeno na dve polovici: hidroelektrarno Mokrice in jedrsko elektrarno z odlagališčem nizko- in srednje radioaktivnih odpadkov (NSRAO). Polovica makete, ki predstavlja hidroelektrarno Mokrice, izpostavlja ključne ekološke in tehnične značilnosti, vključno z dvema ribjima stezama in sončno elektrarno, nameščeno na levi strani struge, pri čemer je voda vizualno prikazana s smolo. Jedrska polovica vključuje odstranljivo odlagališče NSRAO, ki obiskovalcem omogoča vpogled v razporeditev sodov skozi prerez. Vsi deli so bili izdelani s 3D-tiskanjem, sestavljeni, pobarvani in dopolnjeni z detajli ter nameščeni na leseno podlago s kovinsko podporo. Čeprav maketa ni bila namenjena natančni tehnični reprodukciji, učinkovito prikazuje kompleksne energetske infrastrukture, kar omogoča javnosti boljše razumevanje, ozaveščanje in spodbujanje dialoga o nizkoogljičnih tehnologijah. Kombinacija aditivne proizvodnje, interaktivnih elementov in podrobne predstavitve okolja kaže vrednost maket kot orodij za izobraževanje, demonstracijo in promocijo trajnostnih energetskih rešitev.

JOURNAL OF ENERGY TECHNOLOGY
Vol. 18, No. 3, pp. 177–186, November 2025

Professional Article

IDENTIFICATION AND QUALITATIVE ANALYSIS OF LIQUID EFFLUENTS IN THE SPATIAL PLANNING STAGE OF A NEW NUCLEAR POWER PLANT PROJECT

PIA FACKOVIČ VOLČANJK, KLEMEN DEBELAK, KAJA ZUPANČIČ, TOMAŽ ŽAGAR
GEN energija, Krško, Slovenia
pia.fackovic@gen-energija.si, klemen.debelak@gen-energija.si, kaja.zupancic@gen-energija.si, tomaz.zagar@gen-energija.si

CORRESPONDING AUTHOR
pia.fackovic@gen-energija.si

Submitted
10. 11. 2025

Accepted
28. 11. 2025

Published
28. 11. 2025

Abstract The planned JEK2 will generate liquid effluents during normal operation, categorised as radioactive and non-radioactive. The radioactive releases will consist mainly of tritium, while other radionuclides will be present only in trace amounts, all below the authorised limits. The non-radioactive effluents, originating from the cooling and service water systems, will be treated by filtration, pH adjustment and biocide neutralisation. The thermal load of the non-radioactive emissions to the Sava will be minimal to negligible levels, as the cooling tower serves as a heat sink. All the effluents from JEK2 will be managed by design through defence in depth: multi-barrier treatment and holdup, controlled storage and release, and comprehensive monitoring under the ALARA principle to keep the impacts low and within the strict regulatory limits.

Keywords
JEK2,
Nuclear power plant,
liquid effluents,
radioactive discharges,
non-radioactive discharges,
ALARA principle,
effluent management,
regulatory compliance

https://doi.org/10.18690/jet.18.3.177-186.2025

University of Maribor Press

1 Liquid Effluents from Nuclear Power Plants

The operation of nuclear power plants (NPPs) involves inherently the generation of liquid effluents, that must be carefully managed carefully to ensure environmental and radiological protection. These effluents arise from various plant systems and processes associated with power production, maintenance and auxiliary operations. Depending on their origin and characteristics, they are classified into three categories: radioactive, non-radioactive and thermal discharges. Each category requires specific collection, treatment and monitoring measures, to maintain compliance with the regulatory limits and prevent environmental contamination [5].

The radioactive liquid releases originate primarily from the Reactor Coolant System (RCS), the Chemical and Volume Control System (CVCS), the Liquid Waste Processing System (LWPS), and auxiliary systems where small quantities of radioactive substances are transferred to the water streams. The activity of these liquids results from fission products, neutron activation of the structural materials and nuclear reactions, with tritium (^{3}H) as the main radionuclide formed. The collected liquids are treated through filtration, ion exchange, evaporation and decay storage, and subsequently discharged under the strictly controlled and authorised conditions defined in the environmental permit [4], [5], [9].

The non-radioactive effluents originate primarily from the plant's circulating and auxiliary water systems. These waters may contain the dissolved minerals, metals and chemical additives used to maintain the system`s integrity. Prior to discharge, the non-radioactive effluents are treated through processes such as filtration, chemical adjustment, or sedimentation, to ensure compliance with the environmental regulations and permit requirements [2].

The thermal discharges represent another category of effluents. During operation the reactor core generates heat through fission, which is transferred to the primary and secondary cooling systems. This residual thermal energy must be dissipated to a heat sink to maintain safe operation. Depending on the plant design, the heat may be released to a water body, or to the atmosphere via cooling towers [2].

The management of all the liquid and thermal effluents is an integral part of a nuclear power plant's safety and environmental protection strategy. The International and National Regulations require continuous monitoring, optimisation in accordance with the As Low As Reasonably Achievable (ALARA) principle, and transparent reporting to ensure that the discharges remain within the authorised limits, and to ensure the protection of both people and the environment [7].

2 Regulatory Framework

The management of liquid effluents from nuclear power plants is governed by a combination of international recommendations and national legislation, to ensure radiological and environmental protection. The responsibility for implementing and maintaining compliance lies with the plant operator under the supervision of the competent regulatory authorities.

2.1 International Standards and Guidelines

At the international level, the International Atomic Energy Agency (IAEA) provides the basis for regulating radioactive discharges through the safety guide GSG-9: Regulatory Control of Radioactive Discharges to the Environment. This document defines the requirements for national regulatory frameworks, dose constraints for members of the public, and the optimisation of discharges in line with the ALARA principle [5] .

The European Utility Requirements (EUR, Revision E) complement the IAEA guidance by defining the design objectives for new nuclear power plants. For a reference 1500 MWe Pressurised Water Reactor (PWR), the recommended annual limit for liquid radioactive discharges excluding tritium is 10 GBq, while the typical design objectives aim to keep the tritium releases below 40 TBq per year. These limits serve as design reference points, ensuring that the overall radiological impact of normal operation remains negligible [1].

2.2 National Legal Framework in Slovenia

In Slovenia, the effluent discharges are regulated under the Environmental Protection Act (ZVO-2) and the Water Act (ZV-1), which define the emission limits, monitoring obligations and reporting procedures. Environmental permits for non-radioactive discharges are issued by the Ministry of the Environment, Climate and Energy in cooperation with the Slovenian Environment Agency (ARSO). Licensing of the nuclear facility operation, including radionuclide-specific discharges, falls under the scope of the Slovenian Nuclear Safety Administration (SNSA).

For the planned JEK2 unit, all the design and operational requirements will be designed to comply with these regulations, integrating the IAEA and EUR Standards within the national permitting framework.

3 Expected Liquid Effluents from JEK2

During normal operation JEK2 is expected to generate liquid effluents classified as radioactive and non-radioactive. The radioactive effluents will originate primarily from the reactor coolant and liquid waste processing systems, with tritium (^{3}H) as the main radionuclide, while other fission and activation products—such as carbon-14 (^{14}C), strontium-90 (^{90}Sr), cesium-137 (^{137}Cs), and cobalt-60 (^{60}Co)—will be present only in trace amounts [2], [6], [9].

The non-radioactive effluents will originate from the plant's cooling, service and technical water systems, and may contain dissolved minerals, suspended solids, corrosion inhibitors, pH regulators and biocides [2].

The residual thermal energy will be removed via a closed-cycle cooling system with natural-draft wet cooling towers, limiting the heat discharged to the Sava River to a small blowdown flow of approximately 2 m^3/s at around 30 °C. This configuration is expected to dissipate the residual heat effectively while minimising the thermal impacts on the aquatic environment and maintaining efficient plant operation [6].

The effluent management at JEK2 integrates five key elements, which, together, ensure safe, controlled and ALARA-compliant operation:

1. Identification of Radioactive and Non-Radioactive Substances

A comprehensive identification is required of all substances that may occur in liquid effluents during normal operation.

– The radioactive substances include tritium, carbon-14, fission products (e.g., caesium-137, iodine-131) and activation products (e.g., manganese-54, cobalt-60).
– The non-radioactive substances may originate from the cooling and service water systems, and may include dissolved minerals, metals (e.g., copper, iron), the biocides used to prevent biological growth, corrosion-inhibiting chemicals and various organic compounds.

All the radioactive and non-radioactive substances must be evaluated with respect to their properties, potential pathways in the environment, and long-term effects on human health and ecosystems [3], [9].

2. Treatment and Control Systems

Effluent management requires detailed planning and the optimisation of liquid waste treatment systems to minimise environmental impacts. This includes:

– systems for managing liquids containing radionuclides, such as processing, purification and storage in tanks prior to release;
– systems for treating non-radioactive liquids, including filtration, chemical conditioning and dilution before discharge;
– monitoring and automated control, ensuring that the concentrations remain below the authorised limits specified in the operating permit.

It is essential to evaluate the effectiveness of the available technologies and consider the implementation of advanced solutions that reduce the effluent quantities further and improve the overall performance [7].

3. Local Environmental Conditions

The local environmental and hydrodynamic conditions influence the effluent behaviour strongly. For JEK2, this includes a detailed assessment of the Sava River's characteristics, such as dilution factors, seasonal variations in flow and the influence of the existing hydropower plants.
Local ecological components—including fish populations, sediments and riverine habitats—require dedicated evaluation regarding the potential accumulation of radioactive and non-radioactive substances and long-term ecological effects.[6].

4. Regulatory Requirements

During the JEK2 licensing process, the limits for liquid discharges will be established in line with the national legislation and environmental Standards. The Slovenian Nuclear Safety Administration (SNSA) and the Slovenian Environment Agency (ARSO) will determine the authorised discharge limits and monitoring obligations, based on the environmental studies and safety analyses.

These requirements will be included in the operating permit and must be followed strictly by the operator. Continuous monitoring and regulatory reporting will ensure that no unacceptable impacts occur. [5], [8].

5. Impact Modelling and Risk Assessment

Advanced mathematical models will be applied to evaluate the potential impacts of the liquid effluents. This includes:

- calculating the dilution and dispersion of radionuclides and non-radioactive substances in the aquatic environment,
- simulating different operational scenarios, including power variations and start-up/shutdown conditions,
- assessing the long-term effects on the aquatic ecosystem and human health, considering bioaccumulation, potential exposure pathways and the combined effects of multiple pollutants.

Limits for the liquid discharges and water abstraction will be specified in the environmental permit. When these limits are respected, the overall impact of JEK2 on surface waters is expected to remain negligible [6], [8].

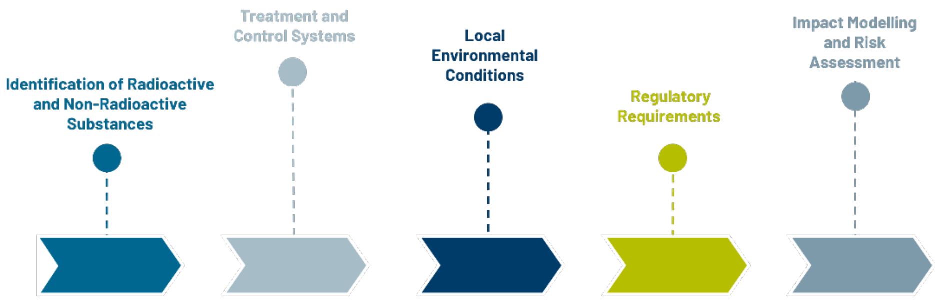


Figure 1: Key Elements of the Liquid Effluent Management at JEK2

Source: own.

This integrated approach ensures that all the liquid discharges remain within the authorised limits, comply with ALARA, and protect both the environment and public health.

Importantly, the effluent releases from JEK2 are planned and managed in a way that minimises the potential influence on the existing Krško NPP (NEK), ensuring that NEK's operational and environmental permits remain fully respected, including under low-flow river conditions.

4 Conclusions

JEK2 is designed to operate with minimal environmental impact while complying fully with international and national safety requirements. Based on the current design documentation for the planned JEK2 unit:

– Effluent management: The planned systems are expected to ensure that all radioactive and non-radioactive liquid discharges can be collected, treated, monitored and released under controlled conditions.

– Thermal impact: The planned closed-cycle cooling system with natural-draft towers is expected to limit the heat discharge to the Sava River effectively (~2 m^3/s at ~30 °C), with a negligible ecological impact [6].

– Safety and regulatory compliance: The design will adhere to the IAEA Safety Standards, EUR design objectives and Slovenian legislation. The effluent generation and release will be optimised according to the ALARA principle, and advanced operational procedures and monitoring systems will ensure safe plant operation with minimal risk of significant radioactive release.

– NEK protection: The effluent management is designed to minimise the impacts on the existing Krško NPP, preserving its operational and environmental permits.

In summary, JEK2's liquid effluent management integrates technological, regulatory and operational measures, to guarantee environmental protection and radiological safety. During normal operation, JEK2 is expected to have a negligible impact on the environment, consistent with the International Standards for safe and sustainable nuclear power generation.

References

[1] EUR. (2016). European Utility Requirements for LWR Nuclear Power Plants (Vol. 2). Brussels: EUR Organisation.
[2] GEN Energija. (2025). Tekočinski izpusti iz JEK2. Krško: GEN.
[3] Huang, H., Liu, Q., Chen, L., Chen, C., & Chen, M. (2021). A statistical evaluation of the liquid effluent release from NPP based on analytical methods with low detection limits. Journal of Radioanalytical and Nuclear Chemistry, 330, 1235–1242.
[4] IAEA. (2005). Environmental and Source Monitoring for Purposes of Radiation Protection (Safety Guide RS-G-1.8). Vienna: International Atomic Energy Agency.
[5] IAEA. (2018). Regulatory Control of Radioactive Discharges to the Environment (Safety Guide GSG-9). Vienna: International Atomic Energy Agency.
[6] IBE. (2025). Pobuda za DPN za JEK2: Predlog pobude. Ljubljana: IBE.
[7] OECD/NEA. (2016). Effluent Release Options from Nuclear Installations: Technical and Regulatory Aspects (NEA No. 3690). Paris: Nuclear Energy Agency, OECD.
[8] U.S. Nuclear Regulatory Commission. (2021). Radioactive Effluents from Nuclear Power Plants – Annual Summary Report. Washington, DC: NRC.
[9] Šavli, S., Křižman, M., Nemec, T., Cindro, M., Stritar, A., Vokal Nemec, B., & Janžekovič, H. (2007). Tritium Liquid Effluents from the Krško NPP. In: Proceedings of the International Conference Nuclear Energy for New Europe (NENE 2007). Portorož, Slovenia: Nuclear Society of Slovenia.

Povzetek v slovenskem jeziku

Identifikacija in kvalitativna analiza tekočinskih izpustov v fazi prostorskega načrtovanja nove jedrske elektrarne. Načrtovana jedrska elektrarna JEK2 bo med normalnim obratovanjem proizvajala tekočinske izpuste, ki so razvrščeni na radioaktivne in neradioaktivne. Radioaktivni izpusti bodo pretežno vsebovali tritij, medtem ko bodo ostali radionuklidi prisotni le v sledovih oziroma bo njihova aktivnost zelo nizka. Neradioaktivni izpusti bodo izvirali iz tehnoloških vod in bodo pred izpustom obdelani s filtracijo, uravnavanjem pH in nevtralizacijo. Toplotne obremenitve reke Save bodo minimalne, saj je za hlajenje predviden hladilni stolp na naravni vlek. Upravljanje tekočinskih izpustov bo zasnovano na principu večplastne zaščite, ki vključuje obdelavo in zadrževanje odpadnih vod, kontrolirano izpuščanje ter celovito spremljanje skladno z načelom ALARA (As Low As Reasonably Achievable), s čimer se zagotavlja minimalen vpliv na okolje in skladnost z zakonodajnimi zahtevami.

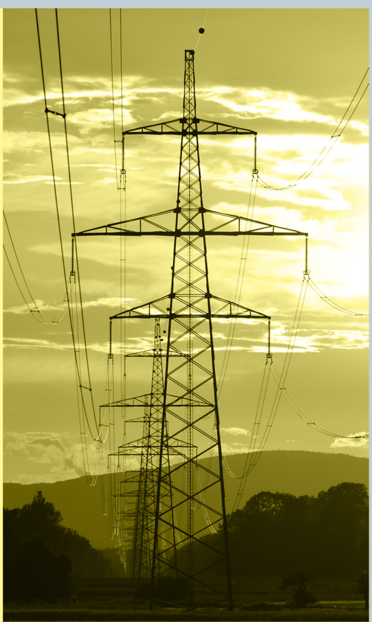
University of Maribor

Faculty of Energy Technology

https://journals.um.si/index.php/jet

jet@um.si

ISSN 2463-7815



JET