Journal of Energy Technology
Vol. 19, No. 1, pp. 31–54, July 2026

Original Scientific Article

Stochastic Modelling of a Low Voltage Distribution Network Operation

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Submitted
9. 6. 2026

Accepted
17. 6. 2026

Published
15. 7. 2026

Abstract Modelling a low-voltage network operation is becoming increasingly challenging. In the past, deterministic methods adopted from sub-transmission and transmission network analysis were sufficient for operators to assess the operating conditions. However, changes in customer behaviour, household appliance structure, and the growing penetration of residential photovoltaics, EV charging, heat pumps, and air conditioning are altering operating conditions in low voltage networks significantly. Given the strongly stochastic nature of individual customer consumption and generation, deterministic methods are becoming less suitable for proper grid operation analysis and development planning. This article presents a simple method for stochastic modelling of a network operation, where customer behaviour is described using a system of histograms. The presented approach enables in-depth investigation of operating conditions in low voltage networks, identifying potential operational extremes that compromise safety of operation, while also quantifying the frequency and probability of such events. Application of the method is demonstrated for a typical rural network.

Keywords
stochastic modelling, low voltage network, probability density, histogram, load flow, development planning

https://doi.org/10.18690/jet.19.1.31-54.2026

University of Maribor Press

1 Introducton

The rapid development of new technologies at both the consumer and industrial levels has driven significant changes in consumer behaviour and introduced new challenges for distribution system operators (DSOs). The growing penetration of residential photovoltaic (PV) installations is transforming consumers into prosumers. In addition, the increasing use of air conditioning and heat pumps, together with the expansion of the residential electric vehicle (EV) charging infrastructure, is altering typical load profiles and transforming operating conditions in low-voltage (LV) networks rapidly (Damianakis, 2025).

To address these challenges, DSOs must analyse network conditions carefully, identify potential weak points, and develop measures to mitigate the negative impacts of these emerging technologies. Accurate modelling of the operating conditions in low-voltage networks is a key component of such analyses. However, modelling the operation of LV networks remains highly challenging, due primarily to the strongly stochastic nature of consumer behaviour.

Several stochastic approaches to modelling consumer behaviour have been proposed, ranging from highly detailed simulations of appliance usage in individual households (Jang, 2016) to various regression-based methods (Yildirim, 2024), neural network-based approaches (Martellotta, 2017), and advanced mathematical frameworks such as copulas (Nelsen, 2006). Each of these methods has a well-defined scope of application in which it performs effectively. However, as in almost every field of science and engineering, there is no such thing as a free lunch (Wolpert, 1997), and from the DSO perspective, these methods each carry inherent practical limitations.

Bottom-up approaches that model appliance usage in detail are extremely complex and difficult to apply at the network scale. Moreover, parameterising such models is highly demanding, making this approach more suitable for the simulation of individual entities, particularly in power quality studies.

Regression-based methods rely on a well-established mathematical framework; however, they require careful tuning and validation of the regression coefficients. Artificial neural network (ANN)-based models can uncover hidden relationships in the underlying data, but their training and validation are often time-consuming. Furthermore, the resulting models are essentially black boxes, making them difficult to interpret directly. Copula-based methods provide an elegant mathematical framework for describing complex dependencies between variables, but they rely on relatively advanced mathematics, and, like regression-based models, require careful parameter tuning with results that are similarly difficult to interpret.

A common drawback shared by ANN, copula and regression-based methods is the need for parameter tuning or model retraining. When the input dataset changes, rebuilding the model typically requires extensive recalibration. This reduces their practicality for industrial applications, where models must be updated regularly to reflect local conditions within the analysed portion of the distribution system.

The purpose of this work is to develop a stochastic modelling framework that enables the construction of statistical models from real-world measurement data with minimal fine-tuning requirements. The framework is designed to support frequent, automated model updates, while remaining easily interpretable and adaptable to represent different customer types.

The proposed model employs a system of histograms to describe the probability density of power consumption for different customer types, while preserving the temporal variability of load throughout the day. The Monte Carlo simulation framework used to generate loading scenarios from these histograms follows the standard principles described in (Kalos, 2008). The performance and usability of the method are demonstrated on a rural low-voltage network. Three scenarios with different customer type compositions are investigated, and the resulting operating conditions are analysed – nodal voltages and line currents. These case studies demonstrate the ability of the proposed method to identify potential violations of operational limits and to quantify the probability of such events.

It should be noted that the current version of the methodology does not preserve the temporal correlation of the generated power samples. As a consequence, the proposed modelling technique is not suitable for tasks involving time integrals of voltages, currents, or powers – for example, the optimisation of the location and sizing of battery energy storage systems (BESS). A different approach must be adopted for such applications (e.g., Beláň, 2022).

This article extends the work presented in (Bahernik, 2026) and (Höger & Bracinik, 2026), providing a more detailed description of the methodology and more comprehensive case studies, including a discussion of the known limitations of the proposed approach.

The rest of the article is organised as follows: Section 2 presents the proposed modelling methodology, Section 3 demonstrates the approach through different case studies, and Section 4 discusses the results, benefits and known limitations of the proposed methodology.

2 Methodology

When assessing current and future operating conditions in distribution networks (both medium and low voltage), the assessment is typically based on load flow calculations, in which the simulation model is parameterised according to the known or expected composition of the customers and their grid connection parameters (reserved capacity, tariff, historical consumption). Historically, such analyses followed the methodology adopted from the sub-transmission and transmission levels. Calculations were performed as deterministic analyses, with the temporal variability of the power profiles represented by typical load curves constructed for different consumer types, usually at an hourly time resolution. This approach was adequate when sufficient safety margins existed to absorb the occasional operational extremes, and when the analysis could focus on the typical, most probable operating scenarios.

However, the growing penetration of residential PV installations, air conditioning units, heat pumps and electric vehicles is eroding these safety margins while increasing the volatility of power profiles. Deterministic methods are no longer sufficient for LV network operation analysis. The main challenge is therefore to develop an assessment method that is both straightforward to implement and capable of providing the required insight into real operating conditions with a suitable level of confidence.

The method must allow as much automation as possible. The stochastic model must be based on existing or easily obtainable data sources, and, given the dynamic changes ongoing in the power sector, must also support frequent and easy model updates. With these requirements in mind, the proposed methodology uses simple histograms to describe the probability density of all the relevant variables and their relationships. This approach avoids the complex retraining required by ANN-based models and the fine-tuning demanded by regression or copula-based solutions, and can be managed by engineers without specialist knowledge of higher mathematics or artificial intelligence. Moreover, histograms are human-readable and directly interpretable, providing transparency at all stages of the simulation process, and facilitating backtracking, debugging and interpretation of the results. To capture the temporal and other relationships between the model variables, the histograms are organised into stacks forming multi-dimensional structures.

2.1 Data preparation and stochastic model

The framework can be implemented using either power profiles or current loading profiles, depending on the modelling approach preferred by the DSO. Measured power or current profiles can be obtained from the existing AMI/IMS infrastructure, or through dedicated measurement equipment installed at selected customer premises. The methodology can accommodate data from sources with different time resolutions; however, to capture the dynamics of real network events, a resolution of 15 minutes or finer is recommended. Sub-minute resolutions are possible but not recommended, as they increase the complexity of the underlying data structures and the number of load flow scenarios that must be solved substantially. A resolution of one or five minutes is currently considered a good balance between model complexity and the ability to capture the relevant network dynamics.

Raw data obtained from different sources must be validated and checked for timing inconsistencies, missing values and data errors. Because the method is not intended to generate time series data, occasional missing samples do not need to be filled artificially, provided the overall dataset contains enough samples to construct a statistically significant model. Data cleaning and re-timing can be performed using the standard tools available in environments such as Python or MATLAB.

After cleaning, the data must be categorised into separate groups according to the consumption type represented by the profile (e.g., standard household, household with a heat pump, household with residential EV charging). Each group is processed separately into a dedicated statistical model. Because the consumption of most consumer types is season-dependent, a separate model must be constructed for each season of interest. The model must also distinguish between workdays and weekends or public holidays, as these categories typically exhibit significantly different behaviour. Depending on the modelling strategy – which may vary considerably with the task and voltage level – the input data can be normalised with respect to the reserved or installed capacity, or, in the case of medium-voltage networks, with respect to the number of customers. In low-voltage networks, normalisation is often unnecessary, and the model can work directly with the absolute values obtained from the measurement system.

The statistical model describes the probability density of the current or power amplitude for each time interval of the day. The consumption data for a given customer type are therefore first divided into time-interval categories according to the required time granularity (e.g. a granularity of 5 minutes yields 288 categories representing a full day). When working with total three-phase loading, samples from different phases of a polyphase measurement system can be aggregated into a single dataset. For each time category, a separate histogram is constructed from the pre-processed dataset. The individual histograms are then stacked together, and, when uniform bin spacing is used across the stack, the system of histograms can be represented as a 2D array, which can be visualised effectively as a heat map. All the histograms are normalised with respect to the total number of samples, so that each histogram value represents a relative frequency in the range 0–100 %, forming a proper probability mass function over the bins. An example of such a model, representing a household with a heat pump during the winter period, is shown in Figure 1.

The framework additionally allows modelling of further relationships – for example, the power factor or reactive power loading as a function of the active power. A similar approach could be applied to other power quality parameters, such as the harmonic content. An example of a P/Q relationship model represented as a heat map is shown in Figure 2.

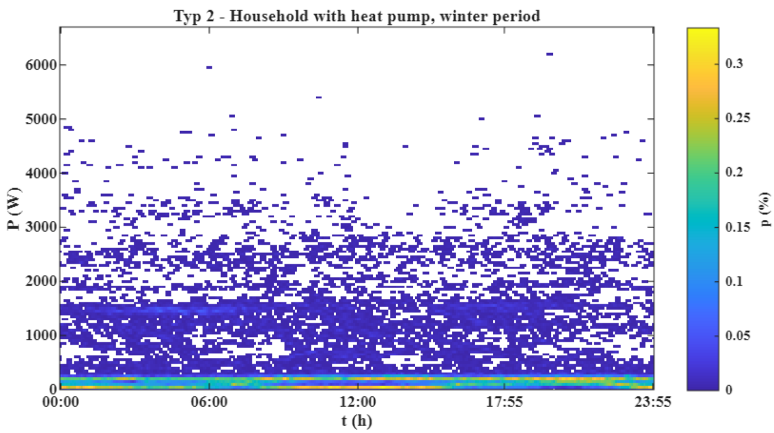

Figure 1: Heatmap representing a statistical model of a household with a heat pump

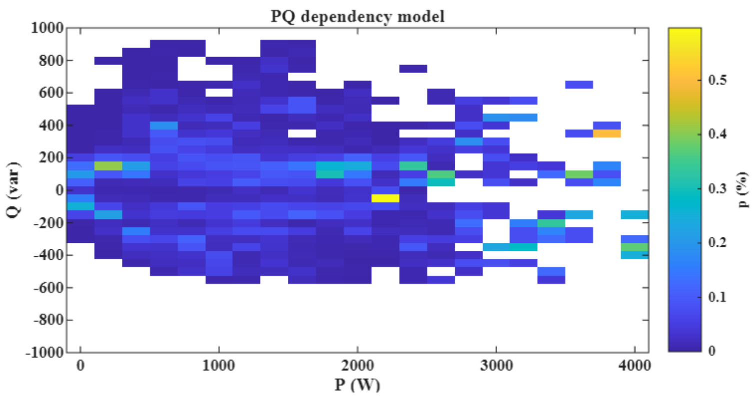

Figure 2: Heatmap representing the relation between the active and reactive power of the load

2.2 Simulation framework

The simulation framework is based on the Monte Carlo methodology (Kalos, 2008). The operating conditions are analysed statistically from the results of a large set of simulated loading scenarios. To enable temporal analysis of the network conditions, the simulation is repeated separately for each time interval at the required granularity. This allows critical time intervals to be identified, and supports tariff management decisions for customers on multi-tariff programmes or subject to mass remote control.

For each time interval, the appropriate histogram is selected from the stochastic load model – corresponding to a single column of the 2D structure – for each consumer type present in the network. A random value of active power or current is then generated for each consumer based on the selected histogram. The flow chart of the proposed framework is shown in Figure 3.

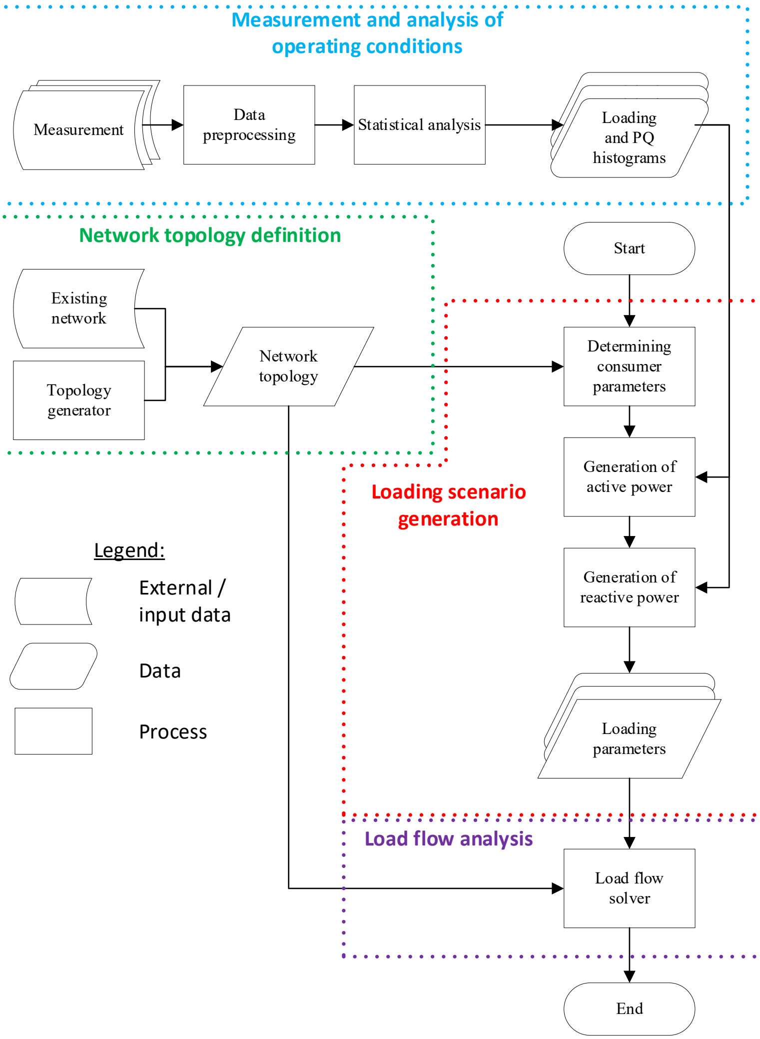

Figure 3: Flow chart of the proposed simulation framework
Source: Baherník et al., 2026

Once the active power for each consumer has been generated, the corresponding reactive power component is determined using the P/Q stochastic model. Based on the generated active power value, the appropriate partial histogram is selected from the P/Q 2D structure, and a reactive power value is sampled from it. This process yields a complex apparent power value for each consumer, together forming a single loading scenario.

Random power values are generated from the histograms using inverse transform sampling applied to the cumulative distribution function (Devroye, 1986), which maps the uniform distribution provided by the built-in random number generator to the desired probability density function describing load behaviour. One inherent limitation of histogram-based generation is the discrete spectrum of generated values; this can be mitigated partially through interpolation between the bin edges.

Each scenario is then solved using a standard load flow solver. For radial LV networks of the type considered here, the Backward-Forward Sweep (BFS) method is particularly well-suited; Gauss-Seidel is a common alternative. The Newton-Raphson method is generally less suitable for medium- and low-voltage networks, due to convergence difficulties arising from the low X:R ratio typical of these systems (Stagg, 2019). For large-scale Monte Carlo simulations requiring the solution of several hundred thousand scenarios, a more computationally efficient implementation based on a GPU-accelerated Jacobi method was developed to support the proposed framework; this solver is described in detail in the referenced work (Höger & Bahernik, 2026).

The number of scenarios required to obtain a statistically significant assessment varies with the network size and customer composition, but is typically in the range of several hundred to several thousand. The framework is therefore computationally intensive, and requires a high degree of algorithmic optimisation, particularly of the load flow solver. Because the individual scenarios are independent, parallel computing techniques can reduce the solution times substantially, with GPU-based implementations offering the best performance.

3 Case study

The introduced methodology was tested on a model of a rural low-voltage network based on a real network from a village located in the northern part of Slovakia. The network has a simple radial structure with two main side branches. The trunk section is 660 m long and the overall length of the network is 1,200 m. The network has 28 nodes and serves 56 customers. To reduce the computational complexity of the model, all the customers are modelled as connected directly to the main part of the network (the customer connection lines are neglected). This simplification reduced the number of nodes in the system from 84 to 28. The network is fed from a 22/0.4 kV distribution transformer (DTS). The single line diagram of the network is shown in Figure 4.

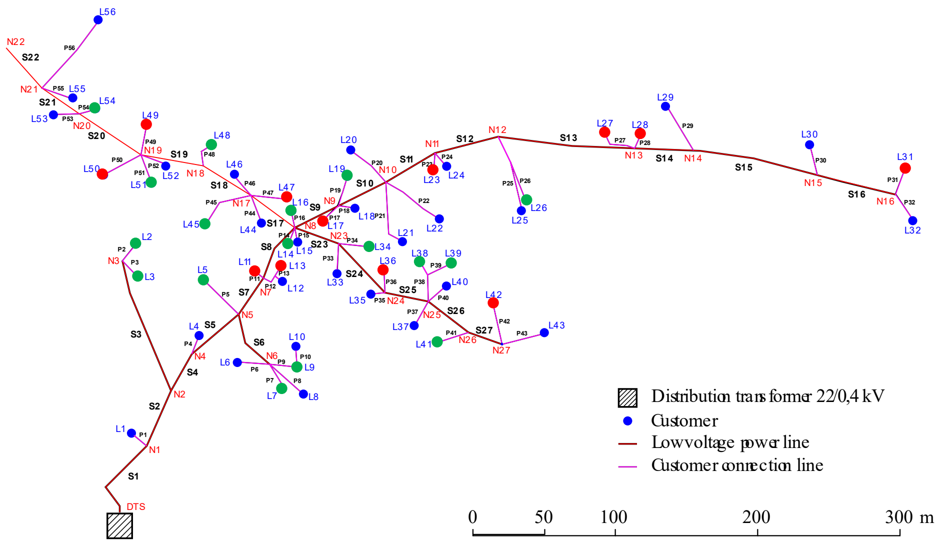

Figure 4: Single line diagram of the low voltage network used for the case study

It should be noted that the node indices used in the results are shifted by +1 relative to the node labels in the single line diagram: node index 1 corresponds to the DTS, so N1 in the diagram corresponds to index 2, N2 to index 3, and so on. Three different kinds of customers were considered:

- Type 1 – regular customers using only small electrical appliances, with no electrical heating or EV charging; this is the reference type,

– Type 2 – customers using a heat pump for space heating and hot water preparation,
– Type 3 – customers using direct electric heating and residential EV charging.

In the single line diagram, an example customer composition is shown, where Type 1 customers are marked in blue, Type 2 customers in green and Type 3 customers in red. The customer composition can be user-specified; the node assignments are generated randomly.

Figure 5 shows heat maps representing the probability distributions of consumption for Type 1 and Type 3 customers. It should be noted that the models represent the early autumn period, during which electric heating is used only sparingly.

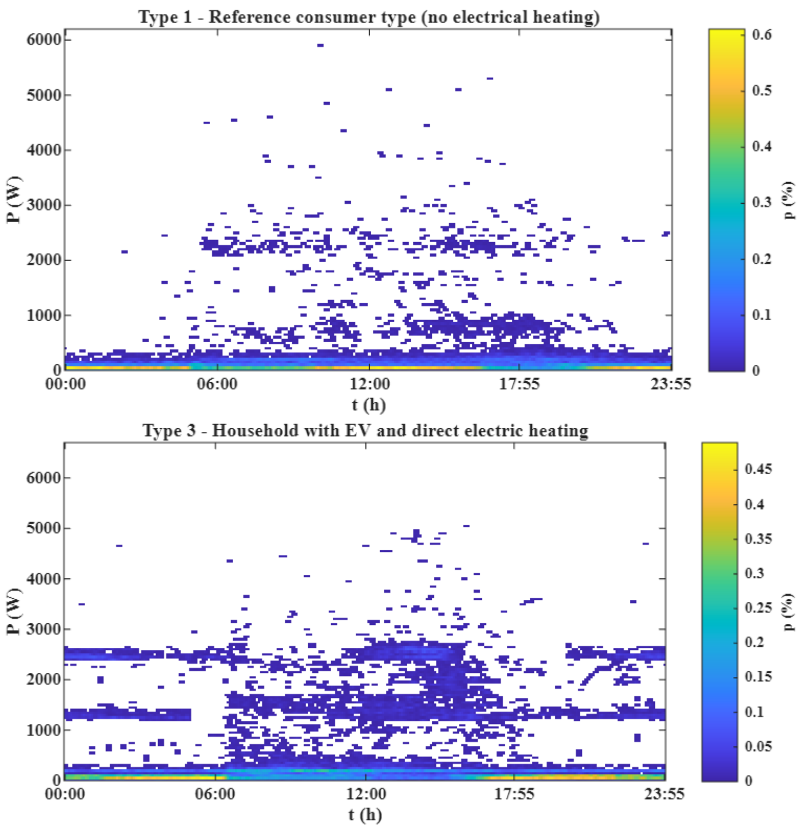

Figure 5: Examples of heat maps representing the probability distribution of consumption for type 1 (top) and type 3 (down) customers for the early autumn period

The histogram system represented by the heat maps was used to generate a loading scenario for each customer randomly. To obtain statistically significant results, a sensitivity analysis was performed by increasing the number of solved operational scenarios gradually while monitoring the spectra of voltages and currents at each node and line section of the network. The analysis showed that at least 1,000 different loading scenarios must be solved for each time interval to achieve statistically significant results. Since the network operation is modelled with a time resolution of 5 minutes, this means that 288,000 different load flow calculations must be solved in total. A highly optimised Jacobi method exploiting massive parallelism and simultaneous solution of multiple scenarios was implemented to support the modelling framework (Höger & Bahernik, 2026). Using a mid-range state-of-the-art GPU (RTX 5070Ti), the solver can complete all 288,000 nonlinear load flow scenarios in under 10 seconds.

Three scenarios were selected to illustrate the potential of the proposed network operation modelling method:

- Reference scenario – all the customers are modelled as Type 1; this scenario reflects the current state of the network,
- Scenario 2 – a customer composition of 50 % Type 1, 30 % Type 2 and 20 % Type 3, autumn period; this represents a scenario close to the operational limits of the network,
- Scenario 3 – a customer composition of 50 % Type 1, 30 % Type 2 and 20 % Type 3, winter period; this represents a scenario in which the operational limits are violated severely due to extensive use of direct electric heating.

3.1 Case 1 – Reference scenario

The reference scenario represents the current state of the network, in which virtually all the customers use gas or solid-fuel heating and hot water preparation. Since the operational limits are never violated in this scenario regardless of the season, a detailed analysis is not presented here. The conditions are illustrated by the histograms of all the nodal voltages and line section currents shown in Figure 6. The voltages are normalised with respect to the nominal voltage; the currents are normalised with respect to the ampacity of each section.

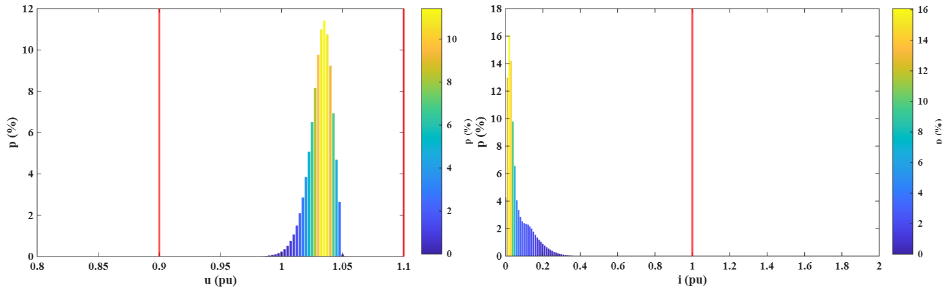

Figure 6: Spectrum of nodal voltages (left) and section currents (right)

As can be seen in Figure 6, neither voltages nor currents violate the operational limits, and a significant safety margin is maintained. The network can therefore be declared safe under any standard operating conditions in this scenario.

3.2 Case 2 – network conditions close to the operational limits

In this case, the customer composition was changed to 50 % Type 1, 30 % Type 2 and 20 % Type 3. The early autumn period was selected for this scenario, so the impact of heat pump and direct electric heating use is relatively limited. The composition was chosen deliberately to demonstrate a network operation close to its operational limits, including occasional violations.

As in the previous case, the analysis begins with a global view of the voltages and currents. The normalised voltage and current spectra are shown in Figure 7.

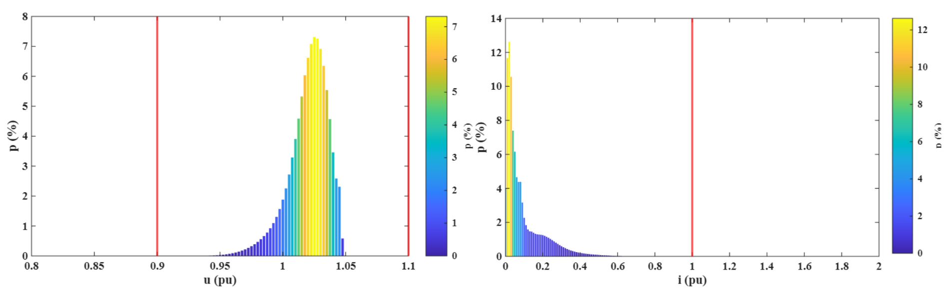

Figure 7: Spectrum of the nodal voltages (left) and section currents (right) for case 2

Compared to the reference scenario, the spectrum of values is wider; however, at first glance there still appears to be a reasonable safety margin. This global view can, however, conceal local anomalies easily. Because the total number of voltage and current samples is very large, with the majority of values well within the limits, the occasional violations are obscured completely. The global view therefore has limited informational value, and a per-element approach is significantly more useful.

In this approach, the voltage and current results are first grouped by the associated network element (node or line section). A separate histogram is then computed for each group. The histograms can be evaluated individually, or, when homogeneous bin spacing is used, stacked to form a 2D structure that can be visualised as a heat map –the stochastic load model is constructed in the same way. This approach allows potentially problematic elements to be identified immediately. The visualisation is configured to display any non-zero probability for a given voltage or current loading category.

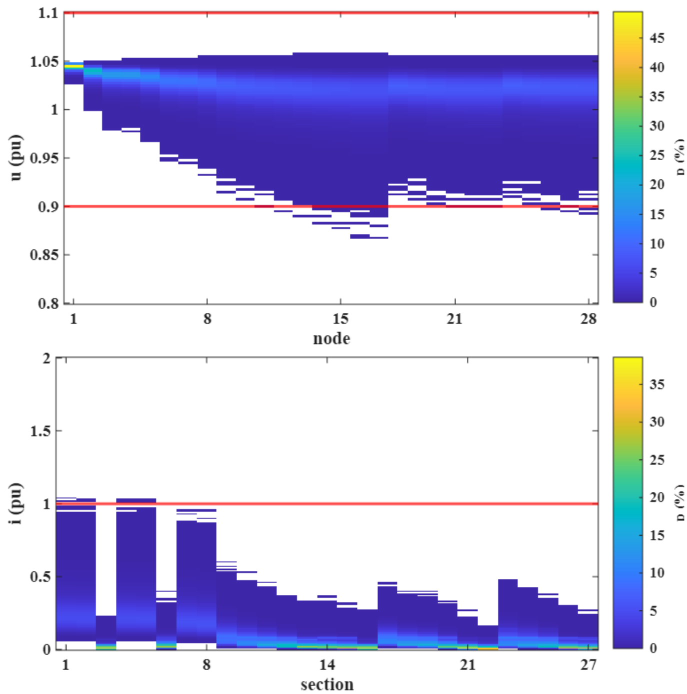

Figure 8: Spectrum of the nodal voltages (top) and section currents (down) for each individual element of the network

From Figure 8 it can be seen that voltage violations of the lower limit were observed at nodes 12–17 and 26–28, with node 17 being the most problematic. These results are consistent with the network topology: node 17 (N16) is located at the end of the trunk, while node 28 (N27) is located at the end of the lower side branch.

A similar situation is observed for the current loading: sections 1, 2, 4 and 5 can be overloaded occasionally and slightly (up to 1.04 pu). The most heavily loaded sections are those at the beginning of the feeder, which supply the majority of the network load (with section 1 carrying the total loading current).

For a network operator, however, the critical information is not merely the existence of possible limit violations, but the ability to quantify their expected frequency and probability. An occasional violation of voltage or current limits may be acceptable from the operator's perspective, not warranting costly network reinforcement. The key benefit of the proposed stochastic modelling technique is precisely this ability to quantify the probability of voltage and current extremes, enabling informed decisions on how to mitigate potential problems.

In this scenario, 9 of the 1,000 simulated daily scenarios contain at least one undervoltage event – that is, at least one node in the network falls below the voltage limit during at least one of the 288 time intervals comprising the simulated day. This corresponds to a probability of 0.9 % per day. Similarly, 3 of the 1,000 simulated scenarios contain at least one overcurrent event anywhere in the network.

The frequency and amplitude of these events are sufficiently low that no immediate network reinforcement is necessary, although the results do indicate that further changes in the customer mix could require a network upgrade within a reasonable time horizon.

Furthermore, because the modelling technique accounts for the time-varying behaviour of consumers throughout the day, it is also possible to analyse the spectrum of voltage and current amplitudes at selected elements across individual time intervals of the day. Figure 9 shows the 24-hour voltage amplitude spectrum for node 17 (N16) and the relative current loading spectrum for line section 1.

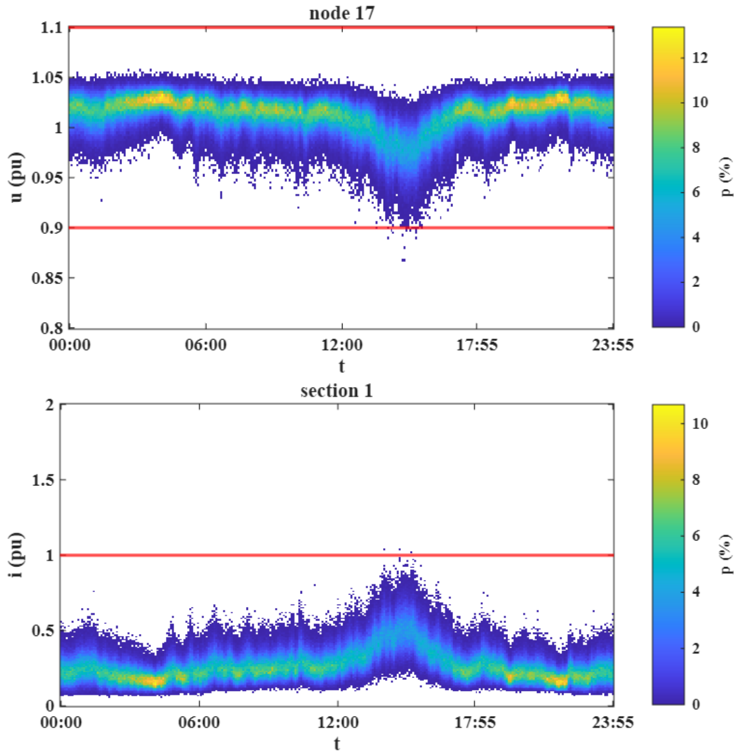

Figure 9: Spectrum of the voltage amplitudes for node 17 (top) and relative current loading for section 1 (down) for individual time intervals of a day

It can be seen that the undervoltage and overcurrent events are concentrated in a limited time window at around 15:00. This consumption peak is caused primarily by Type 3 customers returning home from work, typically between 14:00 and 16:00, and connecting their vehicles to a wall box charger.

This example demonstrates that the proposed methodology enables both spatial and temporal analysis of network operating conditions, including the quantification of the probability and frequency of operational extremes.

3.3 Case 3 – Extensive use of electric heating

The final case uses the same customer composition as Case 2, but simulates operation during the winter season. In this case, both Type 2 and Type 3 customers exhibit significantly increased consumption due to extensive use of electrical heating. As a result, frequent violations of the operational limits are expected.

As shown in Figure 10, a significant number of undervoltage events can be observed, with voltages falling as low as 0.80–0.85 pu; the lowest simulated value was 0.72 pu. A notable number of overcurrent events are also visible, with relative loading reaching 1.4 pu (while some rare events, not visible in the histogram, can reach up to 1.9 pu). Following the same definition as in Case 2, the probability that at least one undervoltage event occurs anywhere in the network during a simulated day is 6 %, while the corresponding probability for an overcurrent event is approximately 2.5 %. Given such high incidence rates, a significant portion of the network is affected – not only the most distant nodes and the first line sections.

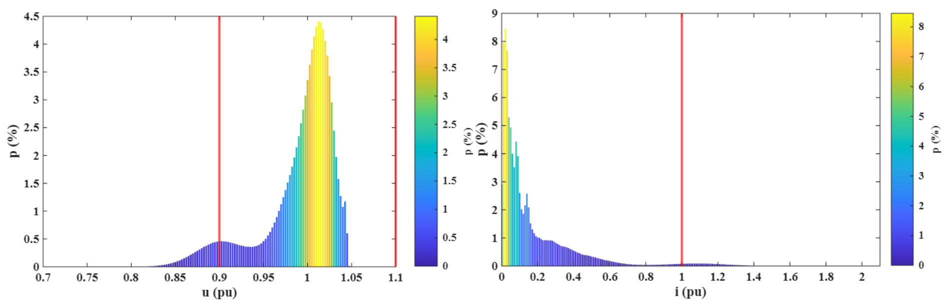

Figure 10: Spectrum of the nodal voltages (left) and section currents (right) for case 3

As shown in Figure 11, most of the network, with the exception of the first five nodes, experiences undervoltage events of varying depth and frequency.

Figure 12 shows that overloading is expected throughout the entire frontal section of the network up to the central junction of the side branches, including the first section of the upper side branch. The most critical elements remain node 17 (N16) and the first section (S1).

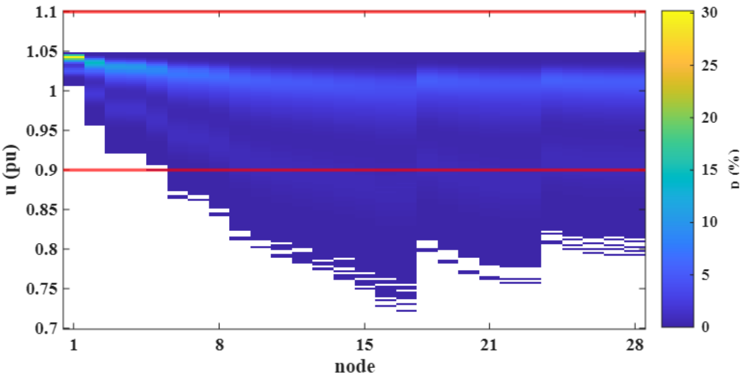

Figure 11: Spectrum of the nodal voltages for each individual element of the network

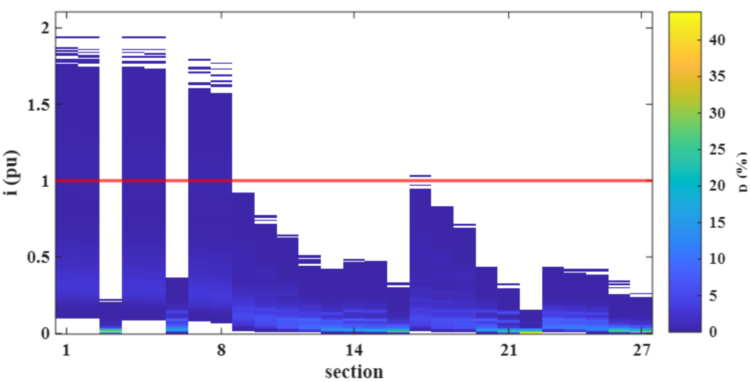

Figure 12: Spectrum of the section currents for each individual element of the network

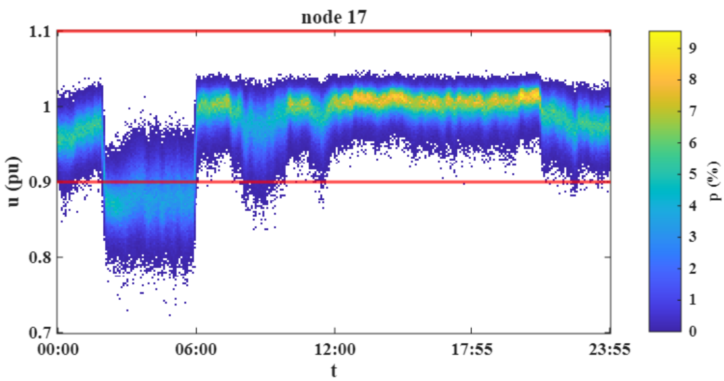

Figure 13: Spectrum of the voltage amplitudes for node 17 for individual time intervals of a day

The temporal analysis is even more revealing in this case. From the heat map in Figure 13, it can be seen that most critical events are concentrated in the interval between 01:00 and 06:00.

A sudden and steep increase in the overall loading causes a significant voltage drop during this interval. The underlying cause is straightforward: the simultaneous activation of the low electricity tariff triggers the start of heat pumps, boilers and direct heaters, which are typically blocked during the high-tariff period. At 06:00, the low tariff ends, shifting the household heating activity to off-peak hours. A similar pattern can be observed in the current loading spectrum shown in Figure 14.

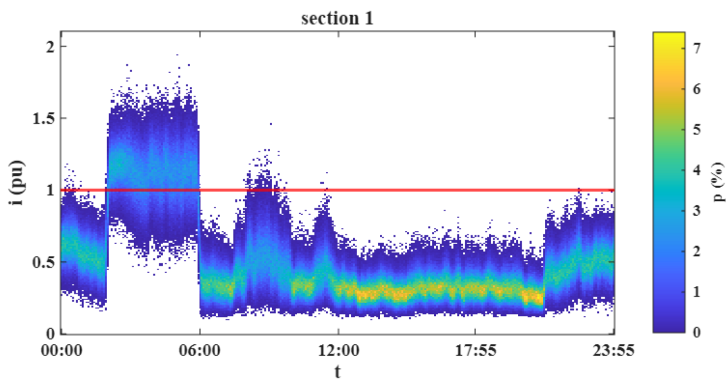

Figure 14: Spectrum of the relative current loading of section 1 for individual time intervals of a day

During this night-time interval, the relative loading of section 1 is between 1.0 and 1.5 pu for most of the period. Under such conditions the network cannot be operated reliably, and the DSO must either limit the number of customers with electrical heating, reinforce the network significantly, or implement a coordination strategy that prevents the simultaneous activation of thermal appliances within the same interval.

In response to these trends, the DSO has already decided to increase the network capacity by splitting the network into two sections: the first covering nodes N1–N16, and the second covering the side branches with nodes N17–N27, connected to the DTS via an aerial bundled cable (ABC) line. This solution increases both the network capacity and security of supply. While this upgrade was not part of the present study, it illustrates that the DSO is aware of the trends identified by the proposed analysis method and has begun implementing preventive measures. Notably, the proposed methodology could also be used directly to evaluate the effectiveness of such upgrades prior to their implementation.

4 Discussion

While the results illustrate the potential of the proposed methodology, an important question is whether the results are consistent with the operation of a real network. A full performance validation would require substantial cooperation with a distribution system operator to obtain detailed customer specifications and access to the infrastructure needed for field measurements. Such field validation is planned for the future; however, no agreement with a DSO for this purpose is currently in place.

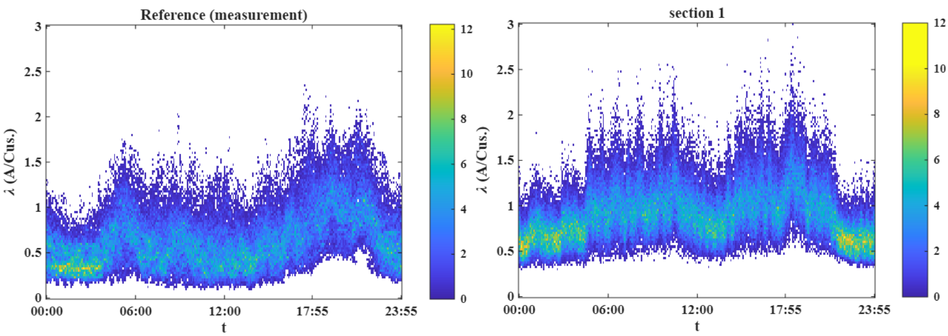

Figure 15: Spectrum of loading factor values for a real distribution transformer (left) and simulation according to Case 1 (right)

Some partial verification of the obtained results is nevertheless possible. To compare the behaviour of a real network with the simulation results, an older dataset of measurements acquired from the secondary side of several 22/0.4 kV distribution transformers can be used, originally collected during a power quality monitoring campaign focused on the power factor and voltage imbalance in low-voltage networks. Because the transformers fed low-voltage feeders with different numbers of consumers, the measured current and power profiles were normalised with respect to the total number of consumers in the corresponding network. This allows the spectrum of loading amplitude to be analysed in a manner similar to that presented in this study (Figure 5), although the analysed variable is not the per-unit current relative to line ampacity, but a loading factor λ related to the number of consumers in the network, with units of A/customer. By recalculating the current loading results for section S1 in terms of this loading factor, the measured and simulated current loading spectra can be compared directly as heat maps.

A direct quantitative validation of the simulated results against measurement data is not possible in this case, as the exact customer composition of the measured network is unknown. A qualitative comparison between the measured relative loading spectrum at the distribution transformer low-voltage output and the simulated spectrum for section 1 is, however, possible. Despite the uncertainty in customer composition, the two heat maps exhibit a comparable overall structure: similar daily loading envelopes, probability density concentration in a similar relative range, and similar scatter characteristics in the upper loading region. This supports the plausibility of the proposed modelling approach, even though precise quantitative agreement cannot be established at this stage.

Beyond this partial verification, the presented case studies demonstrate further the potential of the proposed methodology to analyse operating conditions in the investigated network, and to estimate the impact of changes in consumer behaviour on network performance. The methodology provides both a spatial and a temporal view of network conditions, and allows the probability of technical limit violations to be quantified.

The methodology is based on standard tools of statistical analysis, and does not require the training and verification procedures associated with AI-based models, the mathematical complexity of copula-based or regression models, or the detailed configuration effort of bottom-up approaches based on individual appliance usage modelling.

There are, however, known limitations of the proposed method in its current form. First, the methodology is unable to generate temporally correlated time series. Although the generated dataset resembles a collection of time series superficially, there is no temporal correlation between the individual samples. If the sequence of generated values were treated as a time series, the resulting collection would not exhibit the statistical properties of a real power profile – with the exception of the marginal value spectrum. As a consequence, the methodology is not suitable for problems where the time integral of voltages or currents plays a critical role, such as the dimensioning of battery energy storage systems or the calculation of the thermal loading of equipment. This limitation also has a direct bearing on the interpretation of the violation probabilities reported in Section 3. Although each simulated scenario spans a full day comprised of 288 time intervals, the values generated for these intervals are statistically independent: there is no temporal correlation linking consecutive intervals within a scenario. The reported "per day" probabilities should therefore be understood as the probability that at least one violation occurs somewhere within an independently sampled set of 288 time intervals representing one day rather than as a description of the duration, timing pattern, or sequential persistence of violation events. The probability that a violation occurs at a specific node or section during a specific time interval is unaffected by the absence of temporal correlation, since it depends only on the marginal distribution of that interval; what cannot be inferred from the model is whether, or for how long, a violation might persist once it occurs.

Second, the methodology is currently implemented only for symmetrical (balanced) load flow analysis. In practice, however, the operation of low-voltage networks is typically significantly unbalanced. An extension supporting asymmetric load modelling is currently under development; it will allow the generation of per-phase power profiles that reflect the asymmetry patterns observed in the measured data. This extension was not completed at the time of writing, and will be the subject of a future publication.

Because the method is based on a Monte Carlo approach, it requires an efficient implementation of the load flow solver. Using modern hardware, however, very short solution times are achievable, even for extremely large numbers of scenarios. A nonlinear solver based on the Jacobi method, implemented to exploit GPU computing, can solve several hundred thousand load flow scenarios in a short time. To illustrate the hardware accessibility of the approach: solving all 288,000 scenarios for the investigated network configuration took approximately 70 seconds on a Quadro K2200, an entry-level professional GPU from 2014. On a current mid-range GPU (RTX 5070Ti), the same task is completed in under 10 seconds.

5 Conclusion

Stochastic modelling of low-voltage networks is essential for understanding the current and future bottlenecks in distribution systems, and can support the network planning and decision-making processes of distribution system operators significantly. The presented methodology offers a straightforward, easy-to-implement solution based on a system of histograms describing the probability density distribution of power consumption profiles, including modelling of the power factor. The framework is extensible, with ongoing work targeting the representation of load asymmetry and other power quality parameters.

The primary limitation of the methodology is that it provides only a spectrum of temporally uncorrelated samples, and is therefore not suitable for problems requiring proper time series data, such as battery sizing or thermal analysis. Future work will focus on addressing this limitation, alongside the completion of the asymmetric load flow extension.

Nevertheless, the method is capable of identifying possible violations of operational limits, quantifying the probability and frequency of such events, and providing both spatial and temporal analysis of their occurrence. In this respect it outperforms the traditional deterministic approaches currently used for modelling low-voltage network operation substantially, while remaining accessible to practitioners without specialist expertise in statistical modelling or machine learning.

Acknowledgment

The authors acknowledge the use of an AI-based tool (Claude Sonnet 4.6) for language editing and grammatical correction. No generative AI tools were used in the development of the scientific content, analysis, or conclusions of this article.

Nomenclature

ABC	Aerial Bundled Cable	EV	Electric Vehicle
AMI	Advanced Metering Infrastructure	GPU	Graphical Processing Unit
ANN	Artificial Neural Networks	IMS	Intelligent Metering System
BESS	Battery Energy Storage System	LV	Low Voltage
CDF	Cumulative Distribution Function	PDF	Probability Density Function
DSO	Distribution System Operator	PV	Photovoltaics
DTS	Distribution Transformer Station		

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Povzetek v slovenskem jeziku

Stohastično modeliranje obratovanja nizkonapetostnega distribucijskega omrežja. Modeliranje obratovanja nizkonapetostnega distribucijskega omrežja postaja vse zahtevnejše. V preteklosti so za ocenjevanje obratovalnih razmer zadostovale deterministične metode, povzete iz analiz srednjenapetostnih in prenosnih elektroenergetskih omrežij. Vendar pa spremembe v vedenju odjemalcev, strukturi gospodinjskih električnih naprav ter vse večja razširjenost sončnih fotonapetostnih sistemov, polnjenja električnih vozil, toplotnih črpalk in klimatskih naprav pomembno spreminjajo obratovalne razmere v nizkonapetostnih omrežjih. Ker sta poraba in proizvodnja posameznih odjemalcev izrazito stohastični, deterministične metode postajajo vse manj primerne za ustrezno analizo obratovanja omrežja in načrtovanje njegovega razvoja.

V prispevku je predstavljena preprosta metoda stohastičnega modeliranja obratovanja omrežja, pri kateri je vedenje odjemalcev opisano s sistemom histogramov. Predstavljeni pristop omogoča poglobljeno analizo obratovalnih razmer v nizkonapetostnih omrežjih, prepoznavanje potencialnih obratovalnih ekstremov, ki lahko ogrozijo varno obratovanje omrežja, ter hkrati določa pogostost in verjetnost pojava takšnih dogodkov. Uporaba metode je prikazana na primeru tipičnega podeželskega distribucijskega omrežja.