JOURNAL OF ENERGY TECHNOLOGY
Vol. 18, No. 3, pp. 145–160, November 2025

Original Scientific Article

SYSTEMATIC DERIVATION OF CLARKE AND PARK TRANSFORMATIONS THROUGH VECTOR REPRESENTATION IN THREE-PHASE SYSTEMS

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Submitted
4. 11. 2025

Accepted
20. 11. 2025

Published
28. 11. 2025

Abstract This paper presents a comprehensive pedagogical treatment of Clarke and Park transformations through systematic vector representation, designed specifically for educational purposes in power electronics and motor control courses. While these fundamental transformations are ubiquitous in modern applications–from field-oriented control of AC machines to grid-connected converter control–educational materials often present them either as matrix operations without geometric foundation, or embedded within machine-specific derivations that obscure the general mathematical structure. We address this pedagogical need by progressing systematically from three-phase voltage equations in the time domain to a spatial vector representation in three-dimensional space, deriving the Clarke transformation through explicit geometric projection onto the plane where balanced quantities reside, and, subsequently, deriving the Park transformation as a time-varying rotation of the Clarke frame. The work establishes an amplitude-invariant formulation, provides an explicit geometric interpretation of the 35.26° angle between coordinate systems, and includes worked numerical examples demonstrating complete transformations. This unified pedagogical treatment bridges the gap between practical application and mathematical foundations, providing educators with a complete geometrically intuitive framework suitable for graduate-level instruction and professional development.

Keywords
Clarke transformation, Park transformation, coordinate transformations, power electronics education, motor control, reference frame theory

https://doi.org/10.18690/jet.18.3.145-160.2025

University of Maribor Press

1 Introducton

Three-phase electrical systems constitute the foundation of the modern electrical power infrastructure, from generation and transmission to motor drives and power electronic converters. The analysis, control, and optimization of these systems depend fundamentally on the mathematical transformations that convert voltages, currents, and flux linkages between different reference frames. Among these coordinate transformations, the Clarke transformation ($abc \rightarrow \alpha\beta 0$) and Park transformation ($\alpha\beta 0 \rightarrow dq0$) have emerged as indispensable analytical tools since their introduction in the early 20th century.

1.1 Historical Development and Significance

The theoretical foundations of three-phase coordinate transformations date back to the pioneering work of Robert Park in 1929, who introduced what became known as Park's transformation for analyzing synchronous machines. Park's groundbreaking insight was to transform the stator quantities of a synchronous machine to a reference frame rotating with the rotor, thereby turning time-varying inductances into constants and simplifying the machine's equations. This transformation enabled analytical solutions to problems that were previously intractable in AC machine analysis, and laid the groundwork for modern control theory applications in electrical machines.

Edith Clarke contributed by developing what is now called the Clarke transformation, published in her work on symmetrical components and related topics. Clarke's transformation offered an intermediate step that converts three-phase quantities from the *abc* natural reference frame to a stationary two-axis orthogonal reference frame ($\alpha\beta$), along with a zero-sequence component. This transformation simplifies three-phase system analysis, by reducing the number of variables while maintaining the essential information.

The mathematical elegance and practical utility of these transformations have made them ubiquitous in power systems analysis. Modern applications range from Field-Oriented Control (FOC) of AC machines [1], Space Vector Pulse Width Modulation (SVPWM) for inverters [2], Direct Torque Control (DTC) strategies, grid-connected converter control under unbalanced conditions [3], active power filter design [4], power quality monitoring and assessment [5], fault analysis in power systems [6] to renewable energy system integration [7].

1.2 Educational Motivation and Pedagogical Need

Despite their widespread use across virtually every subdomain of power electronics and electric drives, educational materials often present them in one of two ways: either as matrix operations without a geometric foundation, or embedded within machine-specific derivations that obscure the general mathematical structure. While rigorous derivations exist in advanced textbooks [8], and recent tutorial papers [9] and [10], there remains significant pedagogical value in a unified treatment that:

- Begins from the first principles with general three-phase voltage equations in the time domain;
- Develops 3D spatial vector representation systematically, showing how balanced quantities reside naturally in a specific plane;
- Derives Clarke transformation through explicit geometric projection with clear visualization;
- Connects the Park transformation as a time-varying rotation of the Clarke frame;
- Explains the physical meaning of the key parameters, such as the 35.26° angle and transformation coefficients;
- Provides a worked example suitable for classroom instruction and self-study;
- Addresses practical considerations for digital implementation.

Students in power electronics and electric machines courses benefit from seeing the complete logical progression, from basic three-phase equations to transformation matrices, with explicit geometric interpretation at each step.

1.3 Pedagogical Contribution

This work provides multiple angles of pedagogical contribution for power electronics and electric machines education:

For the Educators: A complete, systematic derivation suitable for graduate-level courses in power electronics, electric machines, and power systems analysis; Geometric visualizations and 3D representations that enhance student intuition; Worked numerical examples ready for classroom use or homework assignments; Practical implementation guidance connecting theory to digital control systems.

For the Students: Clear logical progression from familiar three-phase equations to transformation matrices; Explicit geometric interpretation reducing reliance on memorization; Connection between mathematical operations and physical machine/system behavior.

1.4 Paper Organization

The remainder of this paper proceeds as follows: Section 2 establishes a mathematical framework by presenting general three-phase voltage representations in the time domain, developing a spatial vector form in a three-dimensional space. Section 3 provides a systematic derivation of the Clarke transformation. Section 4 provides a systematic derivation of the Park transformation, explains the frequency translation property, and presents a complete worked numerical example. Section 5 provides a conclusion with a summary of the pedagogical contributions.

2 Mathematical Framework

2.1 Three-Phase Voltage Representations

Three-phase voltages in general form and with arbitrary harmonic content can be expressed as in (1.1):

$$
\begin{aligned}
v_a(t) &= \sum_{h=0}^{\infty} V_{am,h} \cos(h\omega t - hc\frac{\pi}{6} + \varphi_{a,h}) \\
v_b(t) &= \sum_{h=0}^{\infty} V_{bm,h} \cos(h\omega t - hc\frac{\pi}{6} + \varphi_{b,h} - h\tfrac{2\pi}{3}) \\
v_c(t) &= \sum_{h=0}^{\infty} V_{cm,h} \cos(h\omega t - hc\frac{\pi}{6} + \varphi_{c,h} + h\tfrac{2\pi}{3})
\end{aligned} \tag{1.1}
$$

where $V_{am,h}$, $V_{bm,h}$, $V_{cm,h}$ represent amplitudes of the h-th harmonic in each phase, $\omega = 2\pi f$ is the fundamental angular frequency, c is the clock number of the three-phase system, and $\varphi_{a,h}$, $\varphi_{b,h}$, $\varphi_{c,h}$ are the phase angles of the h-th harmonic.

A three-phase system can be represented in a three-dimensional plane, where each phase is associated with each dimension in the way presented in (1.2):

$$\begin{bmatrix} \overrightarrow{V_a(t)} \\ \overrightarrow{V_b(t)} \\ \overrightarrow{V_c(t)} \end{bmatrix} = \begin{bmatrix} \vec{V}_a & \vec{V}_b & \vec{V}_c \end{bmatrix} \begin{bmatrix} \sum_{h=0}^{\infty} V_{am,h} \cos(h\omega t - hc\frac{\pi}{6} + \varphi_{a,h}) \\ \sum_{h=0}^{\infty} V_{bm,h} \cos(h\omega t - hc\frac{\pi}{6} + \varphi_{b,h} - h\frac{2\pi}{3}) \\ \sum_{h=0}^{\infty} V_{cm,h} \cos(h\omega t - hc\frac{\pi}{6} + \varphi_{c,h} + h\frac{2\pi}{3}) \end{bmatrix} \quad (1.2)$$

where $\overrightarrow{V_a(t)}$, $\overrightarrow{V_b(t)}$, $\overrightarrow{V_c(t)}$ represents the resulting phasor of phases *a*, *b*, and *c* in a three-dimensional plane, and $\vec{V}_a$, $\vec{V}_b$, $\vec{V}_c$ represent the ort vectors for each dimension.

The time dependence of the resulting three-phase voltage vector in a three-dimensional plane is obtained by summing up the phasors. The amplitude invariant voltage vector is a three-dimensional plane obtained using (1.3) as:

$$\overrightarrow{V_g(t)} = \sqrt{\frac{2}{3}}(\overrightarrow{V_a(t)} + \overrightarrow{V_b(t)} + \overrightarrow{V_c(t)}) \quad (1.3)$$

Based on (1.2) and (1.3), the balanced harmonic in a three-phase system forms a voltage vector in a three-dimensional space whose trajectory obeys the following rules [11]:

1. Harmonics not divisible by 3 form a circle that lies in a plane (1.4)

$$\overrightarrow{V_a(t)} + \overrightarrow{V_b(t)} + \overrightarrow{V_c(t)} = 0 \quad (1.4)$$

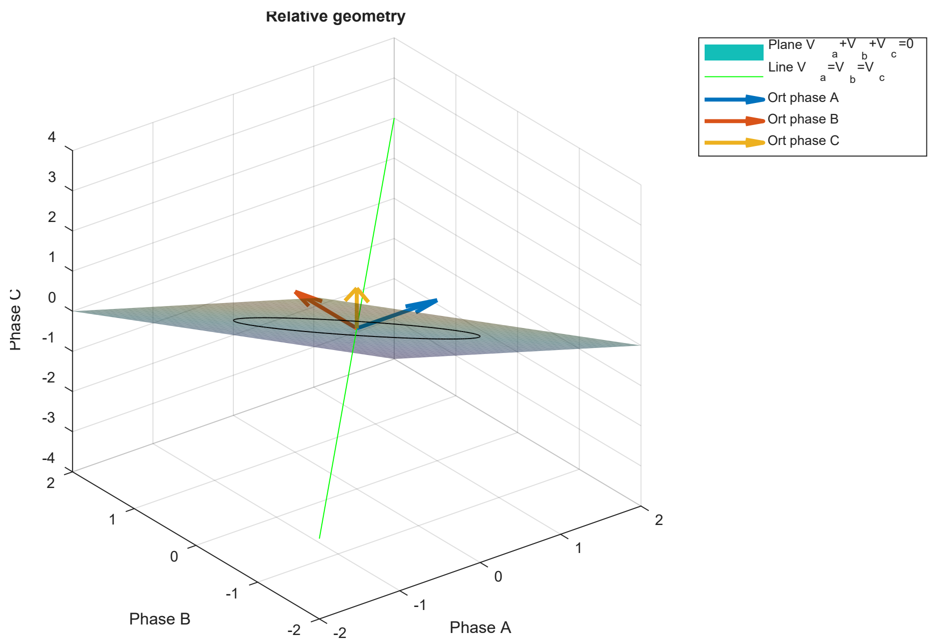
1.1. $3h+1, h \in \overline{1,\infty}$ harmonics have one direction of rotation

1.2. $3h+2, h \in \overline{1,\infty}$ harmonics have the opposite direction of rotation

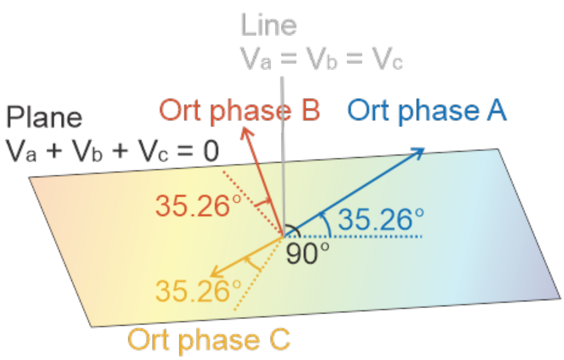
2. Harmonics divisible by 3 ($3h, h \in \overline{1,\infty}$) form a line that lies on a line perpendicular to the previous plane, and has the mathematical description presented in (1.5)

$$\overrightarrow{V_a(t)} = \overrightarrow{V_b(t)} = \overrightarrow{V_c(t)} \tag{1.5}$$

The relative geometry of positive vector orientation, plane (1.4) and line (1.5) is shown in Figures 1 a) and b). The positive vector orientation forms an angle of 35.26° with the plane (1.4). This angle arises naturally from the geometric constrain that three balanced phase vectors must have zero sum. In a three-dimensional *abc* space, consider three-unit vectors representing the phase axes: $\vec{V}_a = [1,0,0]$ (along the phase A axis), $\vec{V}_b = [0,1,0]$ (along the phase B axis), and $\vec{V}_c = [0,0,1]$ (along the phase C axis). A balanced plane is defined as $\overrightarrow{V_a(t)} + \overrightarrow{V_b(t)} + \overrightarrow{V_c(t)} = 0$. The normal vector to this plane is $\vec{N} = [1,1,1]/\sqrt{3}$. The angle θ between the phase A axis ($\vec{V}_a$) and the balance plane equals the complementary angle to that between $\vec{V}_a$ and the normal $\vec{N}$: $\cos(90° - \theta) = \vec{V}_a \cdot \vec{N} = [1,0,0] \cdot [1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}] = 1/\sqrt{3}$. Therefore, $\sin(\theta) = 1/\sqrt{3}$. Using the Pythagorean identity: $\cos(\theta) = \sqrt{2/3}$. The angle between the balanced plane and the phase A, B or C axes can be calculated as $\theta = arctan(1/\sqrt{2}) = 35.26°$. This angle is fundamental to understanding the transformation coefficients, which arise as projections at this specific angle.



a)



b)

Figure 1: Three-Dimensional Geometry of Three-Phase Systems

Source: own.

3 Clarke Transformation

Based on the analysis from the previous section, it can be concluded that, in the three-phase space, the trajectories of the integer multiples of harmonics exist either in the plane defined by equation, (1.4) or on line (1.5). This observation motivates the introduction of a new coordinate system that simplifies the representation of three-phase voltage systems.

We adopt a three-axis coordinate system whose two independent axes lie in the plane (1.4), such that:

- The α-axis is collinear with the projection of the positive orientation of phase A onto the plane (1.4);
- The β-axis lies in the plane (1.4), leading the α-axis by 90°;
- The 0-axis is collinear with the zero-sequence line (1.5).

The graphical representation of this adopted coordinate system is shown in Figure 2. Based on Figure 1. b) the voltage projection of the *abc* system on the newly defined αβ0 system can be formulated as:

$$\vec{V}_\alpha(t) = \sqrt{\frac{2}{3}} \cdot (\vec{V}_a(t)\cdot\cos(0) + \vec{V}_b(t)\cdot\cos(-\frac{2\pi}{3}) + \vec{V}_c(t)\cdot\cos(\frac{2\pi}{3}))\cdot\cos(35.26^\circ)$$
$$\vec{V}_\beta(t) = \sqrt{\frac{2}{3}} \cdot (\vec{V}_a(t)\cdot\sin(0) + \vec{V}_b(t)\cdot\sin(\frac{2\pi}{3}) + \vec{V}_c(t)\cdot\sin(-\frac{2\pi}{3}))\cdot\cos(35.26^\circ)$$
$$\vec{V}_0(t) = \sqrt{\frac{2}{3}} \cdot (\vec{V}_a(t) + \vec{V}_b(t) + \vec{V}_c(t))\cdot\sin(35.26^\circ)$$

(1.6)

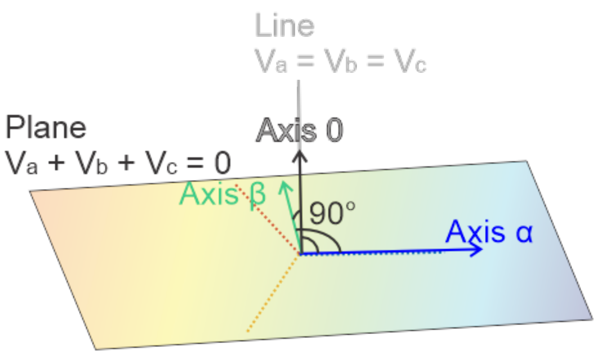


Figure 2: Clarke transformation reference frame (αβ0 system)

Source: own.

By substituting $\cos(35.26^{\circ}) = \sqrt{2/3} \wedge \sin(35.26^{\circ}) = \sqrt{1/3}$, expression (1.6) can be written in matrix form (1.7) with a new matrix defined in (1.8):

$$\begin{bmatrix} \vec{V}_{\alpha}(t) \\ \vec{V}_{\beta}(t) \\ \vec{V}_{0}(t) \end{bmatrix} = \frac{2}{3}\left[T_{\alpha\beta 0,abc}\right]\begin{bmatrix} \vec{V}_{a}(t) \\ \vec{V}_{b}(t) \\ \vec{V}_{c}(t) \end{bmatrix} \tag{1.7}$$

$$\left[T_{\alpha\beta 0,abc}\right] = \begin{bmatrix} \cos(0) & \cos(-\frac{2\pi}{3}) & \cos(\frac{2\pi}{3}) \\ \sin(0) & \sin(\frac{2\pi}{3}) & \sin(-\frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{1.8}$$

The previous expression represents an amplitude-invariant Clarke transformation. Coefficient $2/3$ in (1.7) is often denoted as a scaling factor *K* in the literature, and its variants, along with the area where the specific scaling factor finds application, is presented in Table 1 [11].

Table 1: Clarke Transformation Variants

	Scaling factor *K*	Property Preserved	Typical Applications
Amplitude-Invariant	$2/3$	Peak voltage/current magnitudes	Motor control (FOC), AC current/voltage control
Power-Invariant	$\sqrt{2/3}$	Instantaneous power	Power systems analysis, Power quality
Simplified	$\sqrt{1/3}$	Unit transformation (orthonormal)	Theoretical analysis, Symmetrical components

Source: own

4 Park Transformation

The Clarke transformation establishes a stationary reference frame (αβ0) where two axes lie in the plane of the balanced three-phase quantities. However, in this stationary frame, the balanced AC quantities still appear as sinusoidally varying signals. For many control applications–particularly field-oriented control of AC machines–it is advantageous to work with DC quantities rather than AC signals.

The previous transformation envisages fixed positioning of the coordinate system axes relative to the orientation of the three-phase system axes. The idea of forming a modified coordinate system can be considered, in which two axes lie in the same plane as αβ, but the orientation of the axes changes over time — the axes rotate with some angular velocity $\hat{\omega}$. Of the two axes that lie in the same plane as the αβ axes, we define the *d*-axis and the *q*-axis in the plane to lead by 90° relative to *the d-axis*. As in the case of the 0 αβ system, let the 0-axis remain, oriented perpendicular to the newly formed *dq* plane. A graphical illustration of the coordinate system is shown in Figure 3.

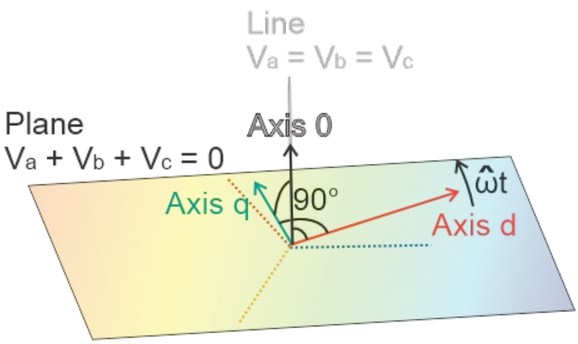


Figure 3: Park transformation reference frame (*dq*0 system)
Source: own.

In the same way as for the Clarke transformation, the voltage projection on the newly defined *d* and *q* axes can be formulated in matrix form as in (1.9) and (1.10):

$$\begin{bmatrix} \vec{V}_d(t) \\ \vec{V}_q(t) \\ \vec{V}_0(t) \end{bmatrix} = \frac{2}{3}\left[T_{dq0,abc}\right]\begin{bmatrix} \vec{V}_a(t) \\ \vec{V}_b(t) \\ \vec{V}_c(t) \end{bmatrix} \tag{1.9}$$

$$\left[T_{dq0,abc}\right] = \begin{bmatrix} \cos(\hat{\omega}t) & \cos(\hat{\omega}t - \frac{2\pi}{3}) & \cos(\hat{\omega}t + \frac{2\pi}{3}) \\ -\sin(\hat{\omega}t) & -\sin(\hat{\omega}t - \frac{2\pi}{3}) & -\sin(\hat{\omega}t + \frac{2\pi}{3}) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \tag{1.10}$$

Or, in the case when the transformation to the *dq*0 system is done from the αβ0 system, the transformation matrix has the form presented in (1.11) and (1.12):

$$\begin{bmatrix} \vec{V}_d(t) \\ \vec{V}_q(t) \\ \vec{V}_0(t) \end{bmatrix} = \begin{bmatrix} T_{dq0,\alpha\beta0} \end{bmatrix} \begin{bmatrix} \vec{V}_\alpha(t) \\ \vec{V}_\beta(t) \\ \vec{V}_0(t) \end{bmatrix} \tag{1.11}$$

$$\begin{bmatrix} T_{dq0,\alpha\beta0} \end{bmatrix} = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) & 0 \\ -\sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1.12}$$

The expressions (1.9) and (1.10), or (1.11) and (1.12) represent an amplitude-invariant Park transformation. Different values of the scaling factor *K* presented in Table 1 can be applied as in the case of the Clarke transformation. Physical interpretation of the Park transformation:

- When the *dq* frame rotates at the same frequency as the αβ voltage/current vectors (synchronous rotation), the projections onto *d* and *q* axes become constant (DC);
- The *d*-axis is, typically, aligned with a meaningful reference (e.g., the rotor flux in FOC);
- The *q*-axis represents the component in quadrature (90°) to the reference.

The Park transformation also acts as a frequency translator:

- The fundamental frequency (ω) values are seen as DC in the synchronous rotating frame;
- DC can be seen as a negative fundamental ($-\omega$);
- The 5th harmonic (5ω) in the *abc* frame as the 6th harmonic with the opposite direction of rotation (-6ω) in the *dq* frame (odd harmonics have an opposite direction of rotation in the plane relative to the fundamental harmonic);

- The 7th harmonic (7ω) in the *abc* frame as the 6th harmonic (6ω) in the *dq* frame (even harmonics have the same direction of rotation in the plane relative to the fundamental harmonic);

To solidify understanding, we present a complete worked example demonstrating the transformation of a balanced three-phase system through both the Clarke and Park transformations.

Given System: A balanced three-phase voltage system with fundamental harmonic only:

- RMS voltage: 100 V per phase
- Peak voltage: $V_m = 100\sqrt{2} = 141.42$ V
- Frequency: $f = 50$ Hz
- Angular frequency: $\omega = 2\pi f = 314.16$ rad/s
- Initial phase of the grid voltage vector (at $t = 0$): $\omega t - c\frac{\pi}{6} + \varphi_a = 0$
- Initial Phase of the Park transformation reference frame (at $t = 0$): $\omega t = 0$ (*d*-axis aligned with phase A)

Calculation of grid voltages in the *abc* domain at $t = 0$ using (1.1) is presented in (1.13):

$$
\begin{aligned}
v_a(0) &= 141.42\cos(0) &&= 141.42 \\
v_b(0) &= 141.42\cos(0 - 120°) &&= -70.71 \\
v_c(0) &= 141.42\cos(0 + 120°) &&= 70.71
\end{aligned}
\tag{1.13}
$$

When the amplitude-invariant Clarke transformation, presented in (1.7), is applied to the instantaneous values obtained in (1.13), the following αβ0 values can be obtained, as in (1.14):

$$\begin{bmatrix} \vec{V}_\alpha(0) \\ \vec{V}_\beta(0) \\ \vec{V}_0(0) \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(0) & \cos(-\frac{2\pi}{3}) & \cos(\frac{2\pi}{3}) \\ \sin(0) & \sin(\frac{2\pi}{3}) & \sin(-\frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 141.42 \\ -70.71 \\ 70.71 \end{bmatrix} = \begin{bmatrix} 141.42 \\ 0 \\ 0 \end{bmatrix} \tag{1.14}$$

When the amplitude-invariant Park transformation from (1.9) is applied to(1.13), (1.15) can be obtained:

$$\begin{bmatrix} \vec{V}_d(t) \\ \vec{V}_q(t) \\ \vec{V}_0(t) \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(0) & \cos(0-\frac{2\pi}{3}) & \cos(0+\frac{2\pi}{3}) \\ -\sin(0) & -\sin(0-\frac{2\pi}{3}) & -\sin(0+\frac{2\pi}{3}) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 141.42 \\ -70.71 \\ 70.71 \end{bmatrix} = \begin{bmatrix} 141.42 \\ 0 \\ 0 \end{bmatrix} \tag{1.15}$$

5 Conclusion

This paper has presented the comprehensive mathematical derivations of the Clarke and Park transformations, from first principles through systematic vector representation. By progressing from three-phase voltage equations to spatial vectors in a three-dimensional *abc* space, we established the rigorous geometric foundations underlying these ubiquitous transformations.

The Clarke transformation emerges as an orthogonal projection from the three-dimensional *abc* space onto the plane $\overrightarrow{V_a(t)} + \overrightarrow{V_b(t)} + \overrightarrow{V_c(t)} = 0$ where balanced quantities reside. The characteristic 35.26° angle and transformation coefficients derive directly from geometric principles, with the amplitude-invariant ($\sqrt{2/3}$ scaling) formulation derived rigorously. The Park transformation follows as a time-varying rotation of the Clarke frame, creating a frequency translation that converts the AC quantities to DC under synchronous rotation.

Coordinate transformations are not merely mathematical tools, but represent fundamental insights about the geometry of three-phase systems. By understanding these transformations from the first principles through systematic geometric derivation, the students and engineers develop a deeper intuition, that enhances their ability to work with modern power electronic and motor control systems.

Acknowledgment

This work has been supported by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia [Grant Number:451-03-136/2025-03/200102]

References

[1] D. W. Novotny and T. A. Lipo, *Vector Control and Dynamics of AC Drives*. Oxford University PressOxford, 1996. doi: 10.1093/oso/9780198564393.001.0001.
[2] F. Blaabjerg, R. Teodorescu, M. Liserre, and A. V. Timbus, "Overview of Control and Grid Synchronization for Distributed Power Generation Systems," *IEEE Transactions on Industrial Electronics*, vol. 53, no. 5, pp. 1398–1409, Oct. 2006, doi: 10.1109/TIE.2006.881997.
[3] X. Wang, F. Blaabjerg, and W. Wu, "Modeling and Analysis of Harmonic Stability in an AC Power-Electronics-Based Power System," *IEEE Trans Power Electron*, vol. 29, no. 12, pp. 6421–6432, Dec. 2014, doi: 10.1109/TPEL.2014.2306432.
[4] H. Akagi, "Active Harmonic Filters," *Proceedings of the IEEE*, vol. 93, no. 12, pp. 2128–2141, Dec. 2005, doi: 10.1109/JPROC.2005.859603.
[5] A. Testa *et al.*, "Interharmonics: Theory and Modeling," *IEEE Transactions on Power Delivery*, vol. 22, no. 4, pp. 2335–2348, Oct. 2007, doi: 10.1109/TPWRD.2007.905505.
[6] D. Bellan, "Clarke Transformation Solution of Asymmetrical Transients in Three-Phase Circuits," *Energies (Basel)*, vol. 13, no. 19, p. 5231, Oct. 2020, doi: 10.3390/en13195231.
[7] M. Liserre, T. Sauter, and J. Hung, "Future Energy Systems: Integrating Renewable Energy Sources into the Smart Power Grid Through Industrial Electronics," *IEEE Industrial Electronics Magazine*, vol. 4, no. 1, pp. 18–37, Mar. 2010, doi: 10.1109/MIE.2010.935861.
[8] P. Krause, O. Wasynczuk, S. Sudhoff, and S. Pekarek, Eds., *Analysis of Electric Machinery and Drive Systems*. Wiley, 2013. doi: 10.1002/9781118524336.
[9] F. G. Montoya and A. H. Eid, "Formulating the geometric foundation of Clarke, Park, and FBD transformations by means of Clifford's geometric algebra," *Math Methods Appl Sci*, vol. 45, no. 8, pp. 4252–4277, May 2022, doi: 10.1002/mma.8038.
[10] C. J. O'Rourke, M. M. Qasim, M. R. Overlin, and J. L. Kirtley, "A Geometric Interpretation of Reference Frames and Transformations: dq0, Clarke, and Park," *IEEE Transactions on Energy Conversion*, vol. 34, no. 4, pp. 2070–2083, Dec. 2019, doi: 10.1109/TEC.2019.2941175.
[11] F. Filipović, "Advanced Synchronization Algorithms for the Operation Improvement of Renewable Energy Source Grid Inverters," Faculty of Electronic Engineering, Niš, 2025.

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Povzetek v slovenskem jeziku

Transformacije z vektorsko predstavitvijo v trifaznih sistemih. Članek predstavlja celovito obravnavo Clarkove in Parkove transformacije skozi sistematično vektorsko predstavitev, zasnovano posebej za izobraževalne namene na področju močnostne elektronike in vodenja električnih strojev. Čeprav sta ti temeljni transformaciji vseprisotni v sodobnih aplikacijah – od poljsko usmerjenega vodenja AC strojev do vodenja omrežno priključenih pretvornikov – učni materiali pogosto predstavljajo transformacije bodisi kot matrične operacije brez geometrijske osnove, bodisi v okviru izpeljav, vezanih na določene tipe strojev, kar zamegli splošno matematično strukturo. V tem delu se posvetimo tej pedagoški vrzeli z doslednim prehodom od trifaznih napetostnih enačb v časovni domeni do prostorske vektorske predstavitve v tridimenzionalnem prostoru. Clarkovo transformacijo izpeljemo kot eksplicitno geometrijsko projekcijo na ravnino, v kateri ležijo uravnotežene veličine, nato pa Parkovo transformacijo kot časovno spremenljivo rotacijo Clarkovega koordinatnega sistema. Delo vzpostavi amplitudno-invariantno formulacijo, poda jasno geometrijsko razlago 35,26° kota med koordinatnimi sistemi ter vključuje numerične primere z vsemi koraki transformacij. Ta enotna pedagoška obravnava zapolnjuje vrzel med praktično uporabo in matematičnimi temelji ter ponuja pedagoškemu kadru popolnoma geometrijsko intuitiven okvir, primeren za podiplomsko poučevanje in strokovno usposabljanje.