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Impediments to Kindergarten Children Identifying Geometric Shapes

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Izvleček/Abstract

Research shows that children aged 3 to 6 years do not fully grasp the concept of geometric shapes. This paper aims to examine children's intuitive knowledge of triangles and squares. We analysed the effects of distractors on identification and the neglected properties of (non)examples. The purpose of the study was to establish the developmental path in the identification of shapes. It was operationalized by determining types of interfering distractors in shape recognition and properties neglected. The data obtained from individual interviews were processed by the method of statistical and descriptive qualitative analysis. A classification was made of distractors and properties of non-examples affecting identification.

Keywords:
a non-example, distractor, geometrical shape, properties of geometric shapes, kindergarten children.

Ovire pri prepoznavanju geometrijskih likov pri predšolskih otrocih

Raziskave kažejo, da imajo otroci, stari od 3 do 6 let, težave pri razumevanju koncepta geometrijskih likov. Osnovni cilj prispevka je preučiti intuitivno znanje otrok o trikotnikih in kvadratih. Analizirali smo vpliv distraktorjev (netipičnih lastnosti) na prepoznavanje likov in neupoštevane lastnosti protiprimerov. Namen raziskave je bil ugotoviti razvojne stopnje otrok pri prepoznavanju likov. Ugotavljali smo vrste distraktorjev in neupoštevane značilnosti pri prepoznavanju likov. Podatke, pridobljene z intervjuji, smo obdelali s statistično in deskriptivno kvalitativno analizo. Oblikovali smo klasifikacijo distraktorjev in značilnosti protiprimerov, ki vplivajo na prepoznavanje likov.

Ključne besede:
protiprimer, distraktorji, geometrijski lik, lastnosti geometrijskih likov, predšolski otroci

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Introduction

The preschool age is a period when basic concepts of geometric figures are constituted (Clements et al., 1999). Those concepts are built on the perceptual similarities of objects in children's immediate surroundings and personal experience (Koleza and Gianissi, 2013). The development continues as children manage to single out and recognize the properties, and finally, to identify geometric shapes based on definition (Satlow and Newcombe, 1998). During this period, children can perceive the properties but still fail to understand which properties play a key role in identifying certain shapes (Clements et al., 2018; van Hiele, 1986). For example, if a triangle is not represented with a horizontal base, children do not identify it and do not perceive it as a triangle (Satlow and Newcombe, 1998). It is considered that recognition of properties and their connection with shapes are the key aspects in the development of geometric thinking (Clements et al., 1999).

Intuition has also been shown to play an essential role in mathematical processes of thinking (Fischbein, 1987). Children's cognitive processes are intuitive, primarily based on practical experience, and heavily influenced by emotions. Consequently, their understandings are frequently deeply ingrained and resistant to external adjustments (Žilkova et al., 2019). Through the process of developing concepts of geometric shapes, there is an interaction of intuitive and formal aspects (Fischbein, 1993). The visual representation of a form allows an instantaneous intuitive response, while geometric concepts are abstract ideas derived from formal definitions. "Very often the intuitive representation is stronger and tends to invalidate the formal conception" (Fischbein, 1987, p. 205). Preschool children recognize geometric shapes by their intuitive aspects, so we are curious about which examples and non-examples of geometric shapes children intuitively (immediately) recognize as such.

Literature Review

Theory of development of geometric concepts

There are several theories about the origin and development of geometric concepts (Đokić and Zeljić, 2017). We will consider van Hiele's theory because of its significant influence on the practice. According to van Hiele's theory, the development of geometric thinking is divided into five progressive levels that lead to formal deductive reasoning (van Hiele, 1986). It develops from the initial, Gestalt visual level, through increasingly sophisticated levels: descriptive and analytical, abstract and relational, the level of formal deduction, and the rigid-mathematical level (Đokić and Zeljić, 2017). The first three levels are crucial for initiating the development of geometric thinking. They describe the processes that lead to an understanding of the relationship between geometric shapes (Bernabeu et al., 2019):

1. Level 1: *visual level.* Children recognize geometric shapes based on their perceptual appearance as a whole, without considering their components. To recognize shapes, they use known prototypes from the environment (e.g., doors for rectangles). Children at this level are able to name shapes and distinguish between shapes of similar appearance.
2. Level 2: *analytical/component level.* Children identify shapes based on properties and can characterize and describe them. At this level, children do not relate these properties to classes of figures.
3. Level 3: *Abstract/rational level* or *level of informal deduction.* Children determine the connections between shapes and can argue their classification. Also, they can detect the properties of a group of shapes based on informative deduction. At this level, hierarchical classification is thought to have been developed.

Clements and Batista (1992) consider that there is a level that precedes the visual level according to van Hiele that is crucial in the process of developing geometric thinking and which is called the *precognitive* or *pre-representative level* (Level 0). At this level, children can only follow a set of visual characteristics of shapes and are unable to identify many common shapes or distinguish figures that belong to the same group.

Other theories also emphasize the importance of early learning of geometry and the introduction of concepts in real-world settings through manipulation and exploration of geometric shapes and materials (Đokić and Zeljić, 2017; Clements and Battista, 1992).

The role of examples and non-examples in developing geometric concepts

Every representation of a geometric concept has certain properties, including some unimportant properties, which we call distractors (Hershkowitz, 1989). As one of the ways to faster and more complete development of geometric concepts, the application of examples and non-examples is often sought.

All examples of a concept must contain its characteristic properties, while distractors can be found only in some representations of the concept. For example, each square has four congruent sides and four equal angles, while its orientation or size is a distractor (Đokić et al., 2020). Hannibal (1999) in his study reveals that many children rely on distractors when trying to distinguish examples from non-examples (see Walcott et al., 2009 for school-age). Burger and Shaughnessy (1986) argue that relying on distractors has elements of visual reasoning. Children on the first level of van Hiele's geometrical thinking (the visual level) tend to include irrelevant properties, such as orientation when classifying or describing geometric figures and tend to reference prototypes when determining figures.

Prototypes, specific types of examples, also play an important role in constituting geometric concepts. Prototypical examples are typical and frequent representations of a geometric concept (Tsamir et al., 2015) that possess the necessary, characteristic properties but also have excessive and unnecessary properties: distractors (Hershkowitz, 1989). For example, an isosceles triangle whose base is horizontally oriented is a prototype of the triangle. In this way, children may have limited understanding of triangles that include only such examples. Children can also include non-examples that visually look like a prototype. Reasoning based on critical properties increases with age (Hershkowitz, 1989).

Tsamir et al. (2008, 2015) argued that some prototypes can be quickly identified as an example of a concept (intuitive examples), while other examples may take longer to identify (non-intuitive examples). They also suggested the possibility that some inappropriate examples are similar, so these are quickly and intuitively recognized as non-examples. Contrarily, non-intuitive or counterintuitive non-examples are those which are not easy to recognize as non-examples of a geometric figure. Clements et al. (1999) suggest that different shapes may have different numbers of prototypes. They claim that a circle and a square have fewer prototypes than rectangles and triangles. Some studies suggest that overexposure to prototypes may hinder the construction of a concept. For example, Kellogg (1980) suggested that prototypes are formed when certain distractors occur frequently in examples, and children begin to associate these distractors with examples of shapes. Wilson (1986) advocated the use of non-examples to minimize the impact of prototypes. By being exposed to non-examples with the same distractors, children can begin to distinguish between basic properties and distractors (Tsamir et al., 2008).

Classification of geometric concepts at the preschool level

The categorization of shapes at the preschool level is done by naming. In this case, naming serves only as an indicator of category creation (Waxman, 1999). Markman (1988) believes that when children hear a new name for an object, they assume that it refers to the whole object, and not to its parts. This corresponds to the visual level (level 0) according to van Hiele in which children first consider the whole shape regardless of its components.

The results of research by Tsamir et al. (2008) showed that about 90% of preschool children were able to identify intuitive triangles, while less than half of children identified non-intuitive triangles. Their findings agree with the results of Clements' research (Clements, 1999). More children successfully identified "non-triangles" than triangles. The identification of non-examples is divided into four categories: 1. reliance on characteristic properties, 2. naming only, 3. consideration of shapes as a whole, and 4. reliance on unimportant properties/distractors. Children usually identified intuitive non-examples only by naming (as soon as they name a square, they know that it does not belong to the category of triangles). When identifying non-intuitive non-examples, children most often relied on the characteristic properties of triangles, although the identification was not always correct.

The construction of geometric concepts is a complex process that includes both visual and descriptive reasoning (Tsamir et al., 2008). "Naming, intuition, and prototypes play a major role in geometric conceptualization" (Ibid, p.85). Previous research has focused on distinguishing different types of examples, including intuitively accepted prototype examples.

Methods

Aim and tasks of the research

The main aim of the research is to determine the developmental path in the identification of geometric shapes among preschool children. For each geometrical shape, we considered its figural concept (Fischbein, 1993). Research shows that images from figural concepts become strong prototypes that dominate the definition process and problem-solving (Satlow and Newcombe, 1998; Wolcott et al., 2009). Based on the aim of the research, the following tasks were formulated:

1. To determine the types of distractors that effectively affect the identification of geometric shapes at a certain age;

2. To determine the properties of geometric shapes that children at a certain age neglect.

Sample

Participants in this study were 151 children of preschool age (3–6 years old) from three Belgrade kindergartens. The sample has the character of an appropriate sample in which children from all four age groups of kindergarten participated ("younger" with 3-year-old children, "middle" with 4-year-old children, "older" with 5-year-old children, and K with 6-year-old children).

Instruments

The survey as a method of data collection was performed by surveying children with a standardized questionnaire. Each child was offered a worksheet that included two complex tasks on both of which the children were asked to identify the two required geometric shapes among the 12 shapes offered (for each shape). At the same time, each geometric shape that they considered to represent a triangle (or square) was to be coloured in, using a different colour. The task that involved recognizing a triangle included the following: 1. one intuitive example, 2. two intuitive non-examples, 3. five non-intuitive examples (geometric shapes with distractors), and 4. four non-intuitive non-examples. The task of identifying squares included the following: 1. one intuitive example, 2. one intuitive non-example, 3. five non-intuitive examples (geometric shapes with distractors), and 4. five non-intuitive non-examples. Given the nature of the geometric shapes whose identification was being examined, there are differences in some distractors and properties that these shapes may possess; thus, the ratio of examples with these properties in tasks differs. The emphasis of the study was placed on the following distractors whose effectiveness in identification was examined: orientation (rotating a geometric shape by a certain number of degrees), size, configuration (shape perception in a complex figure), type of triangle (this distractor can be found only in the case of triangles), as well as on the following non-example properties: curved sides, shape without a single vertex, incomplete borders (only in the case of squares), pattern (with missing shape border), non-intuitive non-example with a different number of sides and non-intuitive non-example with the same number of sides (this property can be found only in the case of squares).

In designing the instrument, we relied on examples of existing research where distractors such as triangle type (Clements et al., 2018), orientation (Tsamir et al., 2008, 2015; Clements et al., 2018) and properties that can be seen in our instrument were used, such as curved side (Tsamir et al., 2015; Clements et al., 2018), shape without a single vertex (Tsamir et al., 2008), as well as intuitive non-examples with a different number of sides (Tsamir et al., 2008, 2015; Clements et al., 2018). In addition to these, we included new properties that we wanted to examine in children: configuration (perceiving shapes in a complex figure) as a distractor, incomplete borders, patterns (with missing shape borders), and non-intuitive non-examples with a different number of sides which have visual similarities with squares (Figure 1).

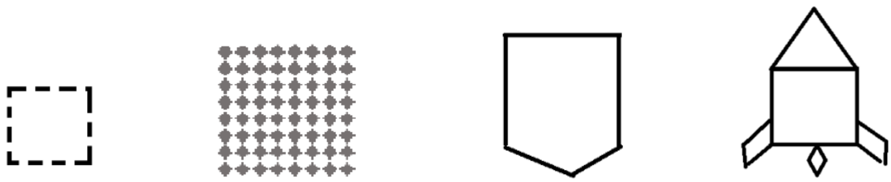

Figure 1: Examples of new properties included in the study: incomplete borders, pattern, non-intuitive non-example with a different number of sides, and configuration.

Data analysis

In order to assess children's ability to identify geometric shapes, a test (paper-pencil) was used. The task for the children was to colour in the specified geometric shape. Each displayed geometric shape received a score of 1 if the child coloured it in, or 0 if the child did not colour it in.

Results

First, we will look at children's ability to identify intuitive examples. The results show that most of the children managed to identify intuitive triangles (92.1%) and intuitive squares (85.5%).

The first research task was to examine the types of distractors that effectively affect the identification of triangles and squares. In Table 1 we present the results for correctly coloured shapes under the influence of a particular distractor. We include only the results from children who gave the correct answer on both tasks and every type of distractor.

Table 1. Children's success in identifying triangles and squares under the influence of distractors

Age	Younger	middle	older	K	In Total
Distractor	%	%	%	%	%
Orientation	48.6	63.9	56.4	73.2	60.9
Size	54.3	63.9	74.4	95.1	72.8
Configuration	0.0	8.3	59.0	73.2	37.1

It can be noticed that the effect of distractors on identification decreases with the increasing age of the children. The Kruskal-Wallis Test confirmed the difference in overall distractor ordinal data between different age groups (H= 46.24(3), p = .000, mean ranks are 47.67; 55.92; 88.56; and 105.87, respectively, for younger, middle, older and K group). Pairwise comparison showed that the younger and middle group of children differ from the older and K group. Test statistics and other statistical information are provided in Table 2.

Table 2. Pairwise comparisons for variable distractor

Comparisons	*Dann's Test*	*SE*	*p value*
younger-older	−40.89	10.00	.000*
younger-K	−58.19	9.89	.000*
Middle-older	−32.65	9.93	.006*
Middle-K	−49.95	9.81	.000*

a. Adjusted p values are provided. Original p values are multiplied by the number of comparisons (Bonferroni's correction)

* <.05

The effects of distractors on the identifications of individual geometric shapes are shown in Tables 3 and 4. Percentages of correct identifications are shown

According to the results from both tables, the same types of distractors have the same effect on both geometric shapes, and this holds true for size and configuration distractors. The Chi-square Test of homogeneity and post-hoc analysis showed that the younger and K group differ from the expected distribution in the case of a triangle and square size. And in the case of configuration as a distractor, each age group differs from the expected count.

Table 3. Children's success in identifying triangles under the influence of distractors

Age	Younger	middle	older	K	In total
Distractor	%	%	%	%	%
Orientation	62.9	88.9	79.5	92.7	81.5
Size	68.6	77.8	89.7	97.6	84.1
Configuration	11.4	16.7	66.7	80.5	45.7
Type of triangle	68.6	66.7	69.2	85.4	72.8

Table 4. Children's success in identifying squares under the influence of distractors

Age	"younger"	"middle"	"older"	K	In total
Distractor	%	%	%	%	%
Orientation	65.7	69.4	69.2	75.6	70.2
Size	60.0	75.0	79.5	95.1	78.1
Configuration	14.3	11.1	69.2	75.6	44.4

Tables 5 and 6 give insight into the statistical information. Although success in orientation distractor has same distribution across age groups considering overall results (both shapes together) ($\chi 2(3)$ = 5.293, p = .152) when it comes to triangle orientation the younger group significantly differs from other groups (Table 6).

Table 5. Post-hoc for-Chi-square Test about the effect of a distractor on a triangle shape

Age	Orientation			size			configuration		
	ASR[a]	*χ2(1)*	*p value*	*ASR*[a]	*χ2(1)*[b]	*p value*	*ASR*	*χ2(1)*[b]	*p value*
younger	3.23	10.44	.001*	-2.87	8.22	.004*	-4.64	21.56	.000*
middle	1.31	1.73	.189	-1.19	1.42	.234	-4.01	16.05	.000*
older	-0.37	0.16	.713	1.12	1.25	.263	3.05	9.32	.002*
K	2.17	4.70	.030	2.76	7.62	.0057*	5.24	27.46	.000*

[a]Adjusted Standardized Residuals

* less than .006 which is the rigid probability level for chi-square post-hoc analysis

Table 6. Post-hoc for-Chi-square Test about the effect of a distractor on a square shape

Age	Size			Configuration		
	ASR[a]	*χ2(1)*	*p value*	*ASR*[a]	*χ2(1)*	*p value*
Younger	-2.96	8.78	.003*	-4.08	16.68	.000*
Middle	-0.52	0.27	.601	-4.60	21.19	.000*
Older	0.24	0.06	.814	3.63	13.17	.000*
K	3.08	9.50	.002*	4.72	22.25	.000*

[a]Adjusted Standardized Residuals
* less than .006 which is the rigid probability level for chi-square post-hoc analysis

Older children were more successful than younger ones, except in the case of triangle orientation. When it comes to identifying triangles affected by orientation distractor, 4-year-old children (the middle group) were 88.9% successful, and 5-year-olds (the older group) were 79.5% successful, but as we see in Table 5, this is not a significant difference from the expected count ($p > .006$). In the case of squares, an orientation distractor had almost the same effect on identification in 4-year-olds (the middle group, 69.4%) and 5-year-old children (the older group, 69.2%), and overall cross tabulation data do not show a statistical difference between age groups related to the square orientation distractor ($\chi2(3) = 0.938$, $p = .816$).

To conclude, the configuration in which a particular geometric shape is represented together with the same or some other geometric shapes is the distractor with the greatest influence on the identification of geometric shapes. Children in the middle and K age groups were better at discerning triangles than squares, while the younger and older age groups performed better when distinguishing squares (Tables 3 and 4). We assume that at this age, a child is acquainted with the properties of squares, so their attention was focused on these shapes. Also, in the transition from year 5 to 6, a jump in the success of shape identification under the influence of this distractor is noticeable for both geometric shapes.

The distractor that is characteristic only for triangles is its type, whereby only a right triangle was included in our research. The results of the research show that 3, 4, and 5-year-old children (the younger, middle, and older groups) react approximately the same to this distractor (Table 3). The difference is noticeable only with the oldest 6-year-old children (K group), where only 14.6% of children failed to identify a right triangle as a type of triangle, but this is not significant ($\chi2(3)$ = 4.525, p = .210). This result is not surprising since research shows that children get to know the concept of angle through different phases of angle abstraction (Mitchelmore and White, 2000).
The second research task was to analyse the properties that children neglect when identifying triangles and squares. We examined the ability of children to recognize a certain property as a property that does not belong to geometric shapes or to recognize a non-example as a non-example of a figure. The results are shown in Table 7.

Table 7. Children's success in identifying the properties of non-examples of triangles and squares

	Younger	middle	older	K	Total
Properties	%	%	%	%	%
Number of sides	45.7	50.0	79.5	73.2	62.9
"Curved" side	37.1	47.2	66.7	65.9	55.0
Shape without a single vertex	20.0	16.7	23.1	17.1	19.2
Pattern	65.7	80.6	53.8	26.8	55.6

The majority of children unsuccessfully identify the properties of geometric shapes. Actually, only 7.9% percent of children identified every property of both geometric shapes. The Kruskal-Wallis Test for overall ordinal data shows that the distribution of non-example recognition is the same across age groups (H(3) = 5.31, p = .151, mean ranks are 76.76, 72.79, 88.24, and 66.52, respectively, for the younger, middle, older and K age groups).

According to mean ranks, it is noticeable that the most successful was the 5-year-old children (older group), and the least successful were the 6-year-olds (K group). The children best recognized the property of geometric shapes that refers to the number of their sides (Table 7). During the identification, the children mostly neglected the property of these two geometric shapes as a closed broken line. Most children coloured in a geometric shape without a vertex (open broken line), considering it to be a triangle or a square (Table 7). Non-examples that are recognized as non-examples in individual geometric shapes are shown in Tables 8 and 9.

Table 8. Triangle non-example recognition

Properties of a figure	younger	middle	Older	K	In total
	%	%	%	%	%
Different number of sides	54.3	52.8	82.1	75.6	66.9
"Curved" sides	60.0	55.6	79.5	75.6	68.2
Triangle without a single vertex	40.0	33.3	41.0	34.1	37.1
Pattern	88.6	88.9	59.0	46.3	69.5

Table 9. Square non-example recognition

Properties of a figure	younger	middle	older	K	Total
	%	%	%	%	%
Different number of sides	62.9	58.3	94.9	82.9	75.5
Same number of sides	62.9	55.6	89.7	73.2	70.9
"Curved" sides	51.4	55.6	84.6	73.2	66.9
Square without a single vertex	28.6	19.4	30.8	19.5	24.5
Incomplete borders	48.6	30.6	28.2	14.6	29.8
Pattern	68.6	83.3	59.0	26.8	58.3

Properties neglected during identification have about the same effect on both geometric shapes. The difference refers to the properties that are least neglected: for triangles, it is the pattern (Table 8), while for squares it is the different number of sides (Table 9).

As we noted before, the results indicate that the property most ignored by children was the property of a geometric shape as a closed geometric figure in a plane; children neglected a side without a single vertex or incomplete borders in the case of squares (Table 9). The number of sides that characterize the geometric shape was best recognized in the case of squares. Five-year-old children noticed this property best. When it comes to triangles, the pattern without a boundary line is the best-observed property (Table 8).

We sought to explore the developmental path in the identification of geometric shapes, so we ran the Kruskal-Wallis Test to determine whether the distribution for recognition of non-examples of a triangle and non-examples of a square remains the same across age groups. In the case of a triangle, all age groups were equally successful ($H(3) = 1.64$, $p = .650$, mean ranks are 77.31, 72.68, 82.85, and 71.56, respectively, for younger, middle, older and K age groups), whereas in the case of a square, significant differences in mean ranks were found ($H(3) = 7.91$, $p = .048$, mean ranks are 74.89, 71.88, 91.59, and 65.74, respectively, for younger, middle, older and K age groups). The Pairwise comparison indicates only that the K group significantly differs from the older group (Dann's Test = 25.85, *SE* = 9.56, adjusted *p value* is .041). This difference suggests that 5-year olds (the older group) outperformed 6-year-olds (the K group) or to be precise, the oldest group of children were mostly unsuccessful in square non-example recognition.

Finally, we sought to determine whether there was a statistical significance in differences between children's recognition of examples of geometric shapes under the influence of distractors and the recognition of non-examples under the influence of characteristic properties of geometric shapes. The Wilcokson Signed Rank Test ($z = -3.29$, $p = .001$) revealed that children were significantly more successful in recognition of examples under the influence of a distractor ($Md = .71$) than in recognition of non-examples under the influence of characteristic properties ($Md = .60$); however, this difference is considered small ($d = 0.2$) (Štemberger, 2021).

Discussion

The results show that a majority of children managed to identify intuitive triangles; thus, our results coincide with those of Clements (1999) and Tsamir (2008). They were less successful in the identification of intuitive squares in comparison to the results of Clements (1999).

When it comes to identifying intuitive non-examples, in comparison to the results of Tsamir (2008), the success rate of recognition of this type of non-example is lower. In the above study, children recognized the pentagon as one of the non-intuitive non-examples of a triangle at a higher rate than children in our study, who recognized a quadrangle as a non-intuitive non-example of a triangle, and a rectangle as a non-intuitive non-example of a square. Considering the Wilcoxon Signed Ranks Test, we see that children showed slightly better knowledge of the distractors and properties of triangles compared to those for squares. This information was not unexpected, since children are usually first introduced to the properties of the triangle. The configuration was the distractor that most often influenced the successful identification of geometric shapes. The most successful in identifying geometric shapes from a complex figure were 6-year-old children, where as many as 73.2% successfully singled out both a triangle and a square. In two examples, the children were asked to detach a certain geometric shape from a complex configuration - one related to the configuration of the same geometric shapes (two triangles or two squares brought together) and the configuration of different geometric shapes, where one of the mentioned geometric shapes was part of a familiar configuration (a triangle presented as a house roof and a square as part of a rocket). The results show that the configuration of the same shapes had a greater effect. Some children from the sample, coloured the examples in different colours, thus clearly distinguishing the geometric shapes (Figure 2). We believe that this activity of the children was influenced by the formulation of the tasks, and we considered their answers to be correct. In addition, a certain number of children coloured in the entire configuration, both the same and different shapes with the same colour. We did not consider these answers, because this children's activity did not indicate that they could single out the required geometric shape.

Figure 2. Coloured configuration in which geometric shapes are separated

When it comes to the properties of geometric shapes, it is noticeable that certain properties of geometric figures do not follow the ascending development path. The properties of squares and triangles in some cases are better noticed and identified by 3 and 4-year-old children than by 5 and 6-year-old children. For example, incomplete square boundaries were ignored by 51.4% of 3-year-olds, as opposed to 85.4% of 6-year-old children. All 6-year-old children who noticed that incomplete borders were not a property of the square "supplemented" the border, and only then coloured in the shape. Some 5-year-old and 6-year-old children "drew" or "filled in" what was missing for a triangle or square to be "complete" (Figure 3). By supplementing the boundaries so that they represent a closed broken line, we believe the children showed that they understood this property of geometric shapes.

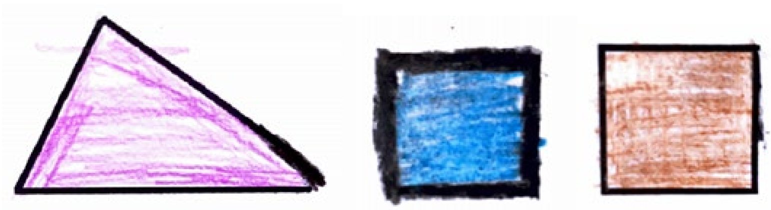

Figure 3. Completed boundary line for geometric shapes

The pattern with a missing boundary line, as one of the properties that do not characterize geometric shapes, was recognized and coloured in by more 5-year- and 6-year-olds than 3-year- and 4-year-old children. Children aged 5-6-coloured in those examples in different colours (Figure 4).

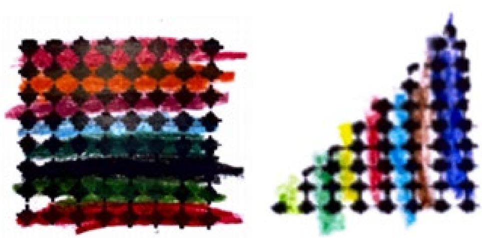

Figure 4. Geometric shapes with patterns marked multi-coloured

Conclusion

Our research focused on distractors that affect identification and properties that preschool children ignore when identifying geometric shapes of triangles and squares.

When it comes to distractors in identification, we conclude that the biggest distractor is the configuration (perceiving shapes in a complex geometric figure). The size of shapes has the least effect on identification as a distractor. Based on these results, we can conclude that with increasing age, the effect of the distractor on the identification of geometric shapes decreases.

According to the results, the properties of geometric shapes are more difficult for children to notice than a distractor. The property of geometric shapes that children first adopt is the number of sides. Children notice this property better in the case of a triangle than with a square. The property that is most often ignored when identifying a plane geometric figure is the closed broken line. In both cases, children were shown a geometric shape without a single vertex, with as many as 80.8% of children neglecting this property. Results also show that children across ages 3 to 6 are equally successful in identifying non-examples of geometric shapes. However, 6-year olds are significantly less successful in the case of square identification under the influence of properties. This should be further investigated. We conclude that there is no progress in the process of shape identification under the effect of non-example properties during age development. Clements and Battista (1992) said that the visual level (Level 1) of thinking is important and could last for years; we add that it also applies to the pre-representative level (Level 0). It is crucial for children to manipulate objects in a real environment and to be geometrically engaged in the early years. This is a critical point for curriculum developers and should be taken into account.

We conclude that children can more accurately identify geometric shapes under the influence of distractors than when some characteristic properties are changed. To sum up, they exhibited better recognition of examples than of non-examples. We presume this to be a consequence of educational practice. The teacher probably provides more examples than non-examples, and such practice needs to be changed.

One limitation of the study is that it focused on only two types of geometric shapes – the ones that children first encounter in the preschool period. The issue should be explored further for other shapes in the future.

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