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Mathematical Causation

Matematična vzročnost

Nekateri trdijo, da nas intuitivne sodbe o matematičnih trditvah vodijo do tega, da verjamemo v matematični platonizem. Toda matematični predmeti platonističnih teorij naj ne bi bili prostorsko časovni in ločeni od sveta, natančneje so nevzročni. To stališče ima težave; če so matematični predmeti ločeni od sveta, se zdi, kot da ne vplivajo na svet. Svet bi bil tak, kot je, tudi če matematični predmeti ne bi obstajali. Poleg tega ljudje, kot je Benacerraf (1973), trdijo, da glede na vzročno teorijo vednosti in glede na njihovo nevzročno naravo nikoli ne bi mogli vedeti o matematičnih predmetih in jih zato ne bi smeli postulirati. Drugje v literaturi ljudje ponujajo neogibne argumente za matematične predmete (npr. Baker, 2009). Zdi se, da se morajo naše najboljše znanstvene teorije sklicevati na razlage, ki kvantificirajo matematične predmete, zato bi jim morali verjeti ravno tako, kot verjamemo v neopazljive predmete znanosti. Potem lahko rečemo, da vemo, da matematični predmeti obstajajo s sklepanjem na najboljšo razlago, ki temelji na naših najboljših znanstvenih teorijah. Toda to nam ne pove, *kakšni* so matematični predmeti, še pomembneje pa je, da še vedno ne odgovori na argumente »ne naredi nobene razlike«. Morda bomo prisiljeni verjeti vanje, vendar nam to ne pove, ali dejansko kaj *naredijo* ali ne. Želim razpravljati o enem od načinov, kako lahko matematični predmeti nekaj *naredijo*. Mislim, da obstaja koristen in informativen način, kako lahko o matematičnih objektih govorimo kot o vzročnih. To storim tako, da razpravljam o primeru matematične omejitve, kot je predlagal Marc Lange (2017). Izpopolnil bom pojem matematične omejitve in govoril o relaciji omejitve na splošno. Nato bom nadaljeval z razpravo o modelih strukturnih enačb in o tem, kako jih lahko uporabimo za predstavitev vzročnih relacij. To še posebej ustreza intervencionističnemu pogledu na vzročno zvezo, ki ga bom opisal, in kako ga lahko uporabimo kot test za ugotavljanje, katere relacije so vzročne. Mislim, da matematična omejitev uspešno prestane ta test. V tem okviru bom razpravljal o posebnem modelu strukturne enačbe, ki predstavlja relacijo omejitve. Ta posebna relacija omejitve je tudi neposredno vzročna. Mislim, da se struktura te relacije naravno preslika na strukturo arhetipskega primera matematične omejitve. Ne samo, da si delita strukturo, ampak se obe relaciji obnašata na enake načine v intervencionističnih obravnavah. To bi nam moralo dati razlog za trditev, da je matematična omejitev vzročna. Za tiste, ki jih zanima epistemologija matematike, lahko to omogoči izogibanje ugovorom v Benacerrafovem slogu; skrivnostno *bi* bilo, kako vemo o nevzročnih matematičnih objektih, toda glede na to, da so dani matematični objekti vzročni, lahko razložimo svojo vednost. Za tiste, ki so naklonjeni ali se zavzemajo za argumente, da to ne naredi nobene razlike, imamo spet odgovor, da so matematični predmeti vzročni, zato *bi* to, če ne bi obstajali, naredilo razliko v svetu.

Ključne besede: intervencionizem, protidejstvenik, model strukturnih enačb, vzročnost, omejitve.

1. Introduction

It is widely accepted in the philosophy of mathematics that mathematical objects, if they existed, would be acausal. Indeed, this acausal nature poses the strongest challenge to mathematical Platonists, since it makes it difficult to explain how we can have knowledge about mathematical objects. Contrary to this orthodoxy, this paper discusses a sense in which mathematical objects might be causal. This is intended as an attempt to tell us not only what sort of things mathematical objects are, but also to explain how it is that they can make a difference and we can gain knowledge about them. This is a controversial view of mathematical objects, but I think there is precedent in the literature for this claim. To try to justify this claim, I will briefly describe the mathematical constraint relation proposed by Marc Lange (2017) and what it is supposed to *do*. To prove that the mathematical constraint can be a causal relation, I will try to show that it fits a typical causal pattern in a structural equation model and that under an interventionist interpretation we can consider the mathematical constraint to behave like a causal relation. In this paper, we build a notion of causality in the tradition of interventionism and structural equation models. After establishing this notion, it will then be applied to a case in mathematics to show that we can view the mathematical constraint relation as a causal relation. I will then discuss some issues that are likely to be raised in response to this view, and respond to them first.

2. Motivations

Given the way we talk about numbers, and certain intuitions, mathematical objects seem to be non-spatiotemporal, acausal entities. Benacerraf and others have raised problems in this regard (Benacerraf 1973 & Liggins, 2010). Given their non-spatiotemporal and acausal nature, it is inexplicable how we can know about mathematical objects. Since knowledge about them seems impossible, our theories postulating them face a serious problem. This argument need not take exactly this form, and instead of referring to knowledge, it may refer to the ability to form justified beliefs about mathematical objects, or it may be granted a warrant to refer to them, and sometimes it is not even expressed in causal terms. The essence of any form of this objection is that without a connection to mathematical objects, the postulating of those objects is in some sense suspect. If this connection is not causal, then it is puzzling exactly what it is. I would respond that there is a connection between mathematical objects and us, and furthermore that connection is a causal one, so it is not puzzling or *ad hoc*. I agree that mathematical objects seem to be non-spatiotemporal, but I don't think that rules out their being causal. It seems plausible that mathematical objects ground the truth of certain statements and facts in the world. I will argue that this justification is a kind of causation, even though mathematical objects are not spatiotemporal; for they act on the world by constraint. To use a classic example: We cannot divide 23 strawberries

(of equal size) equally among 3 people (without them cutting them up) *because* 23 is not divisible by 3 (into whole natural numbers)¹. The mathematical fact constrains the way the world can be, it limits the possible actions mother can take in dividing the strawberries (Lange 2017).² It seems reasonable to say that this is a grounding claim. In a sense, mother cannot divide the strawberries in this way *because of the fact that* 23 is indivisible by 3. I think this is a perfectly natural reading of constraint. Those tempted by Jessica Wilson's (2014) discussion of grounding might want to say that constraint is not identical to big-G grounding, but is instead a kind of small-g grounding relation. Alternatively, those who follow Karen Bennett (2017) might say that constraint, like grounding, is one of the construction relations. However you want to read it, I think it is appropriate and correct to place constraint in this region. I prefer a Bennett-like picture myself. Grounding and causation are two kinds of broad dependency relations, and constraint represents a small area where they overlap. But I think that as long as one accepts grounding in the first place, constraint can be read as neutral across all interpretations, and one need not accept Bennett's picture of building. Returning to the topic of discussion, I argue that grounding by constraint, which is present in mathematical cases like the one above, is a kind of causality because it shares a structure with straightforward causal relations. In structural equation models (SEMs), constrainters in constraint relations occupy the same place as causes in random relations, in interventionist treatments of causality, and so we should be prepared to accept them as causal. I will discuss a basic SEM before moving on to more complicated ones, and then move on to a final example that I believe demonstrates a structure shared by directly causal constraint relations and mathematical relations.

3. Structural Equation Models & Interventionism

SEMs are models designed to represent causal relationships in a clear and simple way, and they allow us to infer causal relationships through statistical data. They have been used with great success in statistical modeling in fields such as physics and economics, and are considered a very useful heuristic. They also have many applications in philosophy, particularly in counterfactual reasoning and the

¹ Of course, 23 is divisible by 3, it just does not yield a whole natural number. For the purposes of this example and simplicity we restrict ourselves to division into natural numbers. This is also the reason we imagine the strawberries cannot be cut up in this scenario. Perhaps the children are unusually picky eaters and will only eat strawberries if they are whole, but still demand an even distribution. Hereafter all these caveats will be assumed and »23 is not divisible by 3« will be referring purely to division into whole natural numbers.

² This is a simple and restricted example but I think it serves to show the principles at play here and demonstrate how one might come to the conclusion that mathematical objects could be causal. There are many other examples of mathematical constraint, and many more complicated ones, these will obviously require slightly different and more complex treatments but those treatments will still be in the spirit of the account proposed in this paper.

analysis of causality. In a SEM, one assigns values to variables and specifies some rules for how the variables interact with each other to model the particular relationship.³ In the following illustrative causal SEMs, *C* and *E* generally stand for cause and effect, respectively. In the following simple case of a stone being thrown against a window, we can see how SEMs look and work:

Variables

C: Whether Suzy throws the stone

E: Whether the window shatters

Structural equations $E=C$

Assignment $C=1; E=1$

Graphical representation



(Wilson 2018: 741)

C causes *E* if interventions that change the value of *C* affect the value of *E* in a particular way. This is the interventionist account of causality and is developed more clearly in some places, e.g. Woodward (2003). Specifically in relation to the causality/grounding unity, this idea is taken up by Alastair Wilson (2018). Wilson suggests that the way interventionism is applied to show that a relationship is causal also works in cases of grounding. For reasons of theoretical unity, among others, he suggests that we simply treat grounding as a kind of causality, a genus of the same kind, so to speak (Wilson, 2001). I am quite sympathetic to this idea, although I am not entirely sure that it works in all cases. For my purposes, I merely want to show that a certain kind of grounding (by constraint) is identical to a certain kind of causation (again, by constraint). I will nevertheless draw on Wilson's examples intended to show that grounding and causation are the same thing in order to build an understanding of interventionism and SEMs. If we look at the above model under an interventionist treatment of causation, we can see how this relationship is causal. We determine whether *C* causes *E* by making interventions on *C*, e.g. preventing its occurrence, and see how this affects *E*. As Woodward

³ A more thorough exploration of SEMs, their use and applications, and their relevance to the topic of causation in philosophy can be found in Hitchcock (2018).

notes, »we can explain what is for a relation between X and Y to be causal by appealing to facts about other causal relations involving I , X , and Y , and counterfactual claims involving the behaviour of Y under interventions on X « (Woodward 2003: 105).

Interventionist explanations of causality are a type of counterfactual explanation that tend to assert that a relationship between X and Y is causal, based on counterfactuals such as »if X had not happened, Y would not have happened«. Specifically, interventionism states whether a relationship between X and Y is causal based on what would happen if an intervention were to occur on X with respect to Y . An intervention is a technical term generally defined by four criteria:

I is an intervention on X iff:

I causes X .

I isolates X from previous causes of X , so that the value of X is fixed by *I* alone.

Any path from *I* to an effect Y goes through X .

I is independent of any variable Z which causes Y and is on a path which does not go through X . (Woodward 2003: 105)

If these criteria are met, then an action can appropriately be called an intervention and the causal relationship can be tested. If an intervention on X in relation to Y causes X and goes on to cause Y , then we can confirm that X causes Y .

We can see how this would work in the case of Suzy. We see Suzy throw the stone, and we see the window break, but she denies that she caused the window to break. Let's imagine that Greg was also standing next to the window at this point, claiming to cast a window breaking spell (Hitchcock, 2018). Now let's imagine that what actually happened is that Greg did not actually cast a spell and the breaking of the window was actually Suzy's fault. We can see how we could recreate the situation and perform interventions to determine this. We could hold fixed Suzy's throwing of the stone while varying Greg's casting of the spell to break the window, and vice versa. Soon we will see dependency patterns emerge between Suzy's throwing and breaking the window. If we are still unsure, we could make further interventions by varying the speed of Suzy's stone throwing, or perhaps varying the obstacles between Suzy and the window. Ultimately, however, we will be able to show that Suzy's relationship to shattering the window is a true dependency relationship, but Greg's casting of the spell is not. Although this is a simple case and in such a situation we would not need to apply the interventionist test to determine the causal relationship, this method can be adapted to more complex scenarios where it is not so clear. In particular, we will see this in the case of mathematics, described later. Interventions and SEMs allow us to determine what the effect depends on, what the cause is in a given situation. In the case

of mathematics, we will see how it appears that this process leads to the result that certain situations are causally dependent on mathematics.

4. Structural Equation Models & Grounding

Now that we have considered interventionism in relation to direct causality and SEMs, we can see how it can be used in relation to grounding relationships. By constructing an appropriate model, we can show what examples of a true dependency relationship exist. I will not define appropriateness rigidly here, as it will vary from case to case; but as an example: If we assume smoking to be the only relevant causal factor for lung cancer, then according to the model above, smoking would not cause lung cancer in interventionist treatments of causality. This is because in some cases people smoke and do not develop lung cancer due to other relevant factors, such as genetic predisposition. When you factor all of these things into a SEM you get the right result, this is where appropriateness comes into play. You might ask why I am modeling grounding relationships with SEMs. What I want to show is that using an appropriate SEM and combining it with interventionism reveals a true dependency relationship. I think this is sufficient to call something causal, but if one is reluctant to do so for various reasons, one might at least be tempted to concede that constraint is a genuine dependency relation. I think such a concession would still allow us to answer the epistemic and »makes no difference« challenges because the constraint relation would be so similar to causality. So I think we would have permission to use it to determine what exists, just as we use causality in the eleatic criterion.

As mentioned earlier, SEMs also model different cases of grounding. Models with the structure of the Suzy case also describe simple grounding:

Simple: Singleton

Variables

C: Whether Socrates exists

E: Whether the singleton set {Socrates} exists.

(Wilson 2018: 741).

Again, we can see how interventionism in this case will expose the dependency relation as genuine by counterfactuals like »if Socrates did not exist, {Socrates} would not have existed«, which are clearly true. In worlds where there is no Socrates, intuition tells us that there would be no singular set containing Socrates. Indeed, it seems difficult to imagine a world in which there is a singular set containing Socrates but no Socrates. This clearly shows that there is some sort of dependency relation at play.

It is worth considering a potential problem here, since answering it will help clarify why interventionism was chosen over other counterfactual explanations of causality. As has been pointed out, interventionism is a kind of counterfactual explanation. If it is true to say »if not- X then not- Y « then it is the case that X caused Y , but what about cases where it is also true to say that if not- Y then not- X ? For example, in the Socrates case above. Finally, it is true to say that if {Socrates} did not exist, then Socrates would not have existed, but we do not think that this is a causal or grounding relation. We want to say that there is a dependence relation at play in the first case, but not in the second. In a sense, the second case tracks the wrong relationship; Wilson (2018: 736) calls such cases wrong-trackers. Interventionism can allow us to avoid wrong-trackers, as Wilson notes: »The distinction between right-tracking and wrong-tracking counterfactuals is then derived in the interventionist framework from a distinction between appropriate and inappropriate causal models. Right tracking counterfactuals are those with antecedents specifying some combination of interventions on model variables in some appropriate model, and with consequents specifying some values for other model variables in that model.« (Wilson 2018: 738). I think this kind of response is related to what we mean by an intervention, and if we look at the criteria that define an intervention above, we can see why. There would be no method of causing (or producing) the nonexistence of {Socrates} that does not go via eliminating Socrates from existence. We cannot satisfy the third criterion, since in such a case the intervention on X with respect to Y would not go via X , there would be a simple causal dependence between the intervention and Y . Perhaps one way to put this idea is in terms of conceivability. It is conceivable that sets do not exist, we can imagine a world without sets. So there are worlds in which Socrates exists, but {Socrates} does not exist. But it could be argued that it is inconceivable for a set to exist without its members, so we cannot imagine an intervention on {Socrates} that leaves Socrates unchanged. It is important to remember that interventionism is not a reductive account of causation, but only an account of when we can call a relation causal or test for it, and in this case no such test can be made. Ultimately, interventionism, in combination with SEMs, can detect genuine dependency relations and distinguish them from false dependency relations.

As mentioned earlier, counterfactual explanations tend to call a relationship causal when it is the case that if X had not happened, Y would not have happened. However, sometimes the relationship between X and Y is causal, even if this counterfactual is false. For example, if there is a »back-up« or preemptive cause, Z , that would have caused Y if X had not happened. Cases like these tend to defeat counterfactual explanations of causation, but this can be accommodated by interventionism and SEMs. This is where the notion of an appropriate model comes in, because including all the relevant factors allows us to see the true dependency relationships in the model. A simple example of this is the case of an *Assassination*.

Two assassins, *A* and *C*, aim their guns at a target, *B*. They both always hit their target, and if they hit their target, it definitely results in death. And when *A* doesn't shoot, for whatever reason, *C* shoots instead. When it happens, *A* fires, and *B* dies. It's clear that *A* caused *B*'s death, but it's not correct to say that if *A* hadn't fired, *B* wouldn't have died because *C* would have fired instead. It seems that a simple counterfactual fails at this point. But the interventionist approach can deal with this. Once you establish the notion of an appropriate model, we can see that simply taking *A* and *B* into account will lead to false causal reports. Interventions in an appropriate model that includes *C* would reveal the genuine causal dependency relationships that are present.

Another such case is presented below:

Early Pre-emption: Marsupials

Variables

C: Whether wombat bites the plant

P: Whether wombat swallows the plant

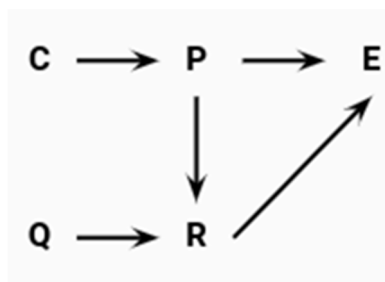
Q: Whether kangaroo sees the plant

R: Whether kangaroo eats the plant

E: Whether the plant is digested

Structural Equations $P=C$ $R=\max(Q-C, 0)$ $E=\max(P, R)$

Assignment $C=1$; $P=1$; $Q=1$; $R=0$; $E=1$



Graphical representation

Wilson (2018: 744)

It is not the case that if the wombat had not bitten into the plant, the plant would not have been digested because the kangaroo was there to eat the plant instead. Let's imagine this model without considering the variables Q and R . We could imagine an intervention being applied to this model, such as muzzling the wombat to see if the plant was digested. As it turns out, the wombat would be muzzled, so unable to bite the plant and therefore unable to swallow the plant, but the plant would be digested because the kangaroo would eat it. It seems that we have shown that the wombat biting the plant does not cause the plant to be digested, but that is the wrong result. What has happened is that we have modeled this situation with an inappropriate model. The model is inappropriate because the kangaroo is also involved. That's why it's so important to include the variables Q and R . We could imagine someone doing the above intervention and noticing the kangaroo's involvement. Given this, we could imagine someone muzzling both the kangaroo and the wombat and observing whether the plant is digested, or releasing the wombat while the kangaroo is restrained and observing what happens to the digest variable. Performing such interventions allows us to determine the dependence relationships and thus establish that the relationships are causal.

At this stage, we have seen how SEMs and the interventionist approach can be used to model and test relationships, allowing us to determine which ones are causal (or at least true dependence relationships). Both are useful tools for assessing causality and, as we will see, also useful for talking about constraint as a relationship.

5. Constraint Relationships

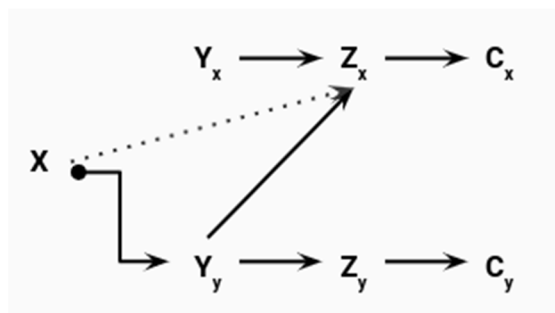
With this understanding of SEMs, we can move on to discuss constraint relations. I will explain a constraint relation, which I hope tells us intuitively that it is causal, before showing that the same structure applies to cases of mathematical constraint. The example we will use is a river. The river flows down a hill and comes to some sort of plateau. There are two potential courses (A and B) that the river could then take from this plateau to flow further downhill. As it happens, the river only flows down one of them, B (perhaps because it is closer or lower). One day a tree falls and blocks the path of river B , causing the river to have to make a new path through another course (A). It seems fairly obvious that the fallen tree has caused the river to flow down through course A . It also seems obvious that the fallen tree has made it impossible for the water to flow down river channel B . This seems then, to be a constraining relationship. The tree has restricted the range of possible »actions« of the water, it has made certain »options« impossible. In the example with the strawberries, the mathematical fact restricts the range of possible distributions, it causes certain distributions of the strawberries to be impossible. The mathematical fact restricts the physical world. The cases are parallel and both seem equally constraining. It is also worth noting that the case is causal

and restrictive, due to the same elements. The cause is the fact that the tree fell (or the event) and the effect is the fact that the water flows as it does (or the event). The constraint is the fact that the tree has fallen, what results from that, the constrained, is the fact that the water flows as it does. Now, of course, you could say that these relationships are not identical, but that instead there are two in play, but I don't see a strong case for that. As I will argue, these relationships behave in the same way under similar interventions, and the patterns of dependence remain the same. Given that they exist between the same elements, we should conclude, on intuition and parsimony grounds, that there is in fact only one relation at play here, perhaps described in different ways, and that this relation is causal and constraining.

Constraint: river

Structural equations: $Y_x = Y_y + 1$, $Z_x = Y_x$, $C_x = Z_x$, $Y_y = X - 1$, $Z_y = Y_y$, $C_y = Z_y$.

Assignment: $X=1$; $Y_x=1$; $Z_x=1$; $C_x=1$; $Y_y=0$; $Z_y=0$; $C_y=0$.



Graphical representation

The dashed line from X to Z_x represents the transitive dependence of Z_x on X , because X ultimately excludes certain other possibilities ($Y_y, \dots Y_L$); it is the exclusion of these other possibilities ($Y_y, \dots Y_L$); it is the exclusion of these other possibilities that causes Z_x . We can see how this abstract structure applies to the river case by assigning variables.

Variables:

X : Tree falls

Y_x : Course A open

Z_x : Water flows through course A

C_x : Consequences

Y_y : Course B open

Z_y : Water flows through course B

C_y : Consequences

In the river case let us suppose that we perform an intervention on X and change its value to 0, i.e., the tree does not fall. We will see that the value of the variable Z_y changes to 1, i.e., the water flows through B and not through A . In this case, it is conceivable that we build a scale model of the tree/river case and make interventions on this model so that we can apply these conclusions to the »real« case. But let us also consider this case with the mathematical variables plugged in:

X : 23 is indivisible by 3.

Y_x : 23 objects divisible between 3 people non-evenly

Z_x : Strawberries are divisible in a particular way

C_x : Consequences

Y_y : 23 objects evenly divisible among 3 groups

Z_y : 23 strawberries evenly divisible among 3 people

C_y : Consequences

Suppose 23 had been divisible by 3, then it would have been possible to divide 23 objects evenly into 3 groups, and furthermore mother would have divided her 23 strawberries evenly among her 3 children. The variable Z_y would have changed in exactly the same way. Likewise, if mother would have divided her strawberries evenly, then she would not have divided them unevenly, i.e., the variable Z_x will also change in the same way as in the case of the tree. Thus, in an interventionist treatment, the fact that 23 is indivisible by 3 caused the 23 strawberries to be indivisible between 3 children. That the course is blocked is the cause of the water flowing down the open one, just as the indivisibility of 23 objects into 3 groups is the cause of the mother distributing the strawberries as she eventually does. An important point to make at this point is that you might want to resist this by placing the cause in this case outside of mathematics. Maybe the cause in this situation is what led to the mother only buying 23 strawberries and not one more (or that she only has 3 children), maybe the cause is some sort of social conditions. I think this is too hasty, because the reason we point to mathematics as *the* explanation (and I want to say cause) is that it explains why such a distribution is impossible in any case, whereas such social conditions would not. Moreover, the social facts may well form an important part of the background conditions that led to the distributional scenario, but it should matter what the cause is within the scenario, to put it that way. We can compare this to the assassination case discussed earlier: The background conditions that led to the assassins being assigned a target are relevant in building the scenario, and we may want to include them as part of the full causes of the scenario. But these background conditions do not thereby prevent the assassin's shooting from being a cause of the target's death. I suggest that we should treat the mathematical case in a similar way. We might want to include the background conditions as part of the relevant causes of the situation, but *in the*

situation the cause we can point to argumentatively is still the mathematical fact, the social facts that matter do not prevent that causal attribution.

6. Possible Problems

6.1 Counterarguments

6.1.1 Intuitive Counterarguments

Before concluding, I would like to discuss some possible problematic differences. In the case of the tree, we are dealing with simple contingent facts that could not easily be true. In the case of mathematics, we are dealing with necessary truths, things that it seems could not have been otherwise. In this case, it is less clear that we are dealing with causality and not some other relation. The answer is that we can use counterpossible statements to take the place of contingent statements in the tree case, such as »if the tree had not fallen«. Counterpossibles like »if 23 had been divisible by 3« can be used in the math case, and we can see if certain situations would have held true in that case. Many people consider counterpossibles to be simply trivially true because of their impossible antecedents, and I think it's worth making some intuitive arguments for why that's premature. We can do this by comparing two counterpossibles:

E. If 23 had been divisible by 3, then 3 would have been a factor of 23.

F. If 23 had been divisible by 3, then 3 would not have been a factor of 23.

It seems clear that *E.* must appear to be true and *F.* must appear to be false. These are not merely trivial statements. Now this non-triviality may admittedly stem from the fact that in *E.* the consequent is merely a reformulation of the antecedent, whereas in *F.* it is a plain contradiction. That may be a good point, but we can still construct non-trivial counterpossibles:

G. If⁴ 23 were divisible by 3, then a calculator would be able to perform the division operation on 23 and 3.

H. If 23 were divisible by 3, then a calculator would not have been able to perform the division operation on 23 and 3.

Again, *G.* should intuitively strike us as true, while *H.* should intuitively strike us as false. Also, compare these two with a further counterpossible:

J. If 23 were divisible by 3, then Paris would be in France.

⁴ Perhaps the antecedent should be lengthened to include »and had mathematicians known this fact« to avoid makes-no-difference style complaints about the inefficacy of mathematical truths on our practice. This would still produce a counterpossible.

A counterpossible such as *J*. seems empty in an important way that *E-H* are not. This intuitive judgement should hopefully show that there is room for the notion that counterpossibilities can have non-trivial truth values. Now it is of course the case that in mathematical cases, as opposed to physical constraints cases, the modality we are dealing with is different. But the abstract structure of these relations is the same, and how they behave when intervened upon is unchanged, which seems to be more important in determining a relation as causal; the modality may be regarded merely as a difference in degree rather than in kind.

6.1.2 Counterpossibles in science

Some people may not be persuaded by so-called »intuitive« counterpossibles like those we discussed above. Simply put, one might simply deny that these have different truth values, they are one and all true, and any appearance to the contrary is mere appearance. However, there are counterpossibles that are not so easily dismissed. Many scientific theories and models appeal to counterpossibles in their explanations and predictions. If these were all and only trivially true, then we might worry that the scientific endeavour is under threat. Moreover, the judgement that such scientific counterpossibles are non-trivial is not at all pre-theoretical or based only on intuition. Instead, such a judgement is based on scientific reasoning. In other words, such examples provide a good amount of positive support for the non-triviality of counterpossibles. Not only do we need to think through and figure out with more than mere intuition that such counterpossibles are non-trivial, but we also need to judge them as non-trivial in order to make progress and predictions.

Numerous examples of such counterpossibilities are offered and discussed by Tan (2019). Not only are there numerous examples of counterpossibles used in science, but they are also used in different ways for different purposes. One area where Tan (2019) focuses on their use is in scientific explanation. He offers an archetypal example of a counterpossibility and discusses why viewing it as a counterpossibility and as non-trivial is the correct judgement. The case offered is:

»If diamond were not covalently bonded, then it would be a better electrical conductor« (Tan, 2019: 40).

Tan claims that this is a scientific explanation for the fact that diamond cannot conduct electricity, while solid carbon in some other forms can. The reason why covalent bonding explains this fact is because covalent bonds do not leave free electrons because they »consume« all the electrons that form the strong bond. In other substances, free electrons enable electrical conductivity (Tan, 2019: 40). The property of poor conductivity that diamond has is caused by these bonds and thus by the microphysical structure. So this counterfactual offers an explanation by highlighting this dependence relationship. But one might wonder if this is indeed a counterpossible, one might wonder if diamond could have been bound differently, and so whether this is just a simple counterfactual. There are two ways to

approach this, we can consider whether something is called diamond because of its microphysical structure or because of its theoretical role in science (Tan, 2019). If we go the first way, we can easily see that this is a counterpossible, because if something is diamond only because of its microphysical structure, then something that had a different microphysical structure would not be diamond. It is a metaphysical necessity that diamond has the structure that it has. So it is metaphysically impossible for diamond to be bound in any other way.

If you go the second way, you might think that we define diamond by its theoretical role, the diamond substance is the substance that makes x , y , and z . But the reason diamond is different from other substances and the reason it does the things it does is because of its microphysical structure. In other words, nothing else could do the things that diamond does without its microphysical structure. Nothing could fill the role of diamond without actually being diamond. So again, in other words, it is metaphysically impossible for diamond to be bound in any other way than it actually is. So it seems that »If diamond had not been covalently bonded...« is a counterpossible. Tan (2019) goes further than this, insisting that this is also counterpossible which is true, and not-vacuously so. For it describes an empirical fact that the poor conductivity of diamond depends physically on its microphysical structure. Thus, science relies on non-vacuous counterpossibles for scientific explanation (Tan, 2019: 42). It is easy to see that this is not an isolated case, as many explanations of why substances have the properties they do will rely on a similar explanatory structure in a scientific context. So it looks like we will need to invoke non-trivial counterpossibles in some cases, so hopefully their use in this paper will not seem so controversial, keeping that in mind.

6.2 *Intelligibility of interventions*

Another possible problem is the intelligibility of interventions: We can understand what it means to stop the tree from falling, but we can't understand what it means for 23 to be divisible by 3. I think this objection is pretty closely related to the last one. It's easy enough to understand counterfactuals like »if the tree hadn't fallen« because we know what this world would be like, and we can imagine ourselves holding everything fixed in this world except for the tree falling. But with counterpossibles like »if 23 had been divisible by 3«⁵ is less clear, and what that world would look like is puzzling. If we change this mathematical fact, then of course people will think we need to change more of the math, there will be a massive wave of changes throughout the world, and the objection will be that there is too much to make sense of to be able to make any statement about the truth or falsity of things in such a world non-trivial. So we could make any arbitrary statements about imagined interventions in mathematical facts, but they will not make sense, and in any case would not allow us to conclude what would lead from

⁵ Or more precisely, »if 23 had been divisible by 3 into whole, natural numbers«.

them because we do not know what else would be true or false in that world. To respond to this kind of worry, I would like to make use of a response offered by Marc Lange (2019) in response to these questions concerning another theory. Lange sees the crux of the problem as being that counterpossibles »spill out« into the world to such an extent that we cannot make sense of what is true and false in this world. He claims that this is the case with many common counterfactuals that people would use. For example, we tend to think of counterfactuals like »if Julius Caesar had lived today . . .« and are happy to say that we can hold this fixed without changing anything. But that's not really the case, because we can ask how come Julius Caesar is alive today, did he time travel? That would require a lot of differences, was it a time travel machine built by humans, did he fall through a wormhole in a sci-fi-esque accident? Maybe instead of time travel it's just that the baby that was born and grew up to be Julius Caesar was actually born in 2007. But if he didn't have the same upbringing (which is probably impossible, considering how much society has changed), then he really isn't Julius Caesar. Lange's point is that although it may seem that ordinary counterfactuals hold when »everything else is fixed«, this is not in fact the case (Lange, 2019). All counterfactuals undulate because the world is fundamentally contiguous. So criticising counterpossibles for this is a weaker criticism than intended, since it also applies to counterfactuals. We may have to do some work to figure out what exactly gets fixed in counterpossible worlds and what remains unchanged, but the fact that this work has to be done is not a problem. Alternatively, it seems that in ordinary rippling counterfactual cases we can simply stipulate that such counterfactuals are non-trivially true or false, so this should be available to us in the counterpossible case as well (Lange, 2019). I think Lange got this right, just because a counterfactual seems easy to understand does not mean it is, and just because a counterpossible seems impossible to understand does not mean it is. Moreover, the literature on impossible worlds is constantly expanding, and much work has been done in this area to show that impossible worlds have non-trivial truth conditions. It may well turn out that all of these accounts are false, but while the debate is still lively I don't think we should jump to that conclusion, and so in the meantime we can make do with such accounts to justify the kinds of counterpossibles and impossible interventions we want.

7. Conclusion

In conclusion, the constraining relation seems to be a real one that operates on the world. Moreover, it is at least arguable that this relation shares a structure with relations that we would normally want to describe as causal. Given this, it seems as if we should conclude that constraint is a causal relation. This means that because mathematical objects are involved in constraint relationships, they are involved in causal relationships. Given this, there seems to be an open causal path available

to us that could allow us to avoid the Benacerraf problem and gain knowledge of mathematical objects, and also to avoid »makes no difference« arguments, because mathematical objects do make a difference, thus allowing us to postulate them. In this contribution we have seen one application of this thinking and how it can give us knowledge of the natural numbers. In a similar vein we can see how one might construct a mathematical version of the pre-emption case we discussed earlier. The fact that 17 does not have 2 as a factor is *because* it is a prime number, but equally even if it had not been prime, it is an odd number, and this also explains why it does not have 2 as a factor. This seems like some pre-emption/overdetermination is at play here (perhaps the explanation could be considered to be in the other direction, with the fact the number is odd being more important, but clearly there will be some sort of pre-emption/overdetermination). Perhaps such an explanation could be of use in a constraint explanation of a variation of the classic cicadas case (Baker, 2005). The reason that 17-year life cycle cicadas do not intersect with predators every 2 years is *because* 17 is prime, but even if 17 had not been prime, it is an odd number. So one might wish to say this non-intersection is overdetermined/pre-empted by mathematical facts. Of course so far we have only considered how natural numbers can be involved in constraint relations, and that is a small part of mathematics. Thankfully, the epistemic advantage of appealing to constraint could give us much more. Certain percolation phase transitions seem to exhibit constraint and this would ultimately grant us knowledge of the real numbers. Lange (2017:7–8) argues that the example of the Königsberg bridges is a constraint explanation. It is a constraint explanation because the arrangement of bridges and landmasses makes it impossible to traverse all the bridges exactly once in a single attempt, called an Eulerian path. To perform an Eulerian path, it must be the case that, considered as a network, either every vertex, or every vertex but two, is touched by an even number of edges, I will refer to this as the Eulerian fact. We can see how we could plug this into the above model to produce a similar constraint explanation.

Variables:

X : Eulerian fact

Y_x : The Königsberg bridge arrangement can be crossed exactly once each in a single trip

Z_x : A given person, P , performs an Eulerian path over the bridges

C_x : Consequences

Y_y : The Königsberg bridge arrangement cannot be crossed exactly once each in a single trip

Z_y : A given person, P , performs a non-Eulerian path over the bridges

C_y : Consequences

What I hope this shows is that the constraint approach to mathematical explanation, treated as a causal relation can give us knowledge of, and a connection to,

further mathematical objects. This provides us with a project and a process going forward, treating constraint as causal, or even as an important dependence relation in its own right, has the potential to give us a path to knowledge about many different areas of mathematics and go a large portion of the way to solving the epistemic problem.⁶

Mathematical Causation

Some argue that intuitive judgments about mathematical statements lead us to believe in Mathematical Platonism. But the mathematical objects of platonistic theories are supposed to be non-spatiotemporal and detached from the world; more precisely, they are acausal. This is problematic because if mathematical objects are detached from the world, then it would seem that they make no difference to the world. The world would be the way it is even if there were no mathematical objects. Moreover, people like Benacerraf (1973) argue that given a causal theory of knowledge, and given its acausal nature, we could never know about mathematical objects and therefore should not postulate them. Elsewhere in the literature, indispensability arguments are made *for* mathematical objects (e.g., Baker, 2009). Our best scientific theories seem to rely on explanations that quantify about mathematical objects, so we should believe in them just as we believe in the unobservable objects of science. We can then say that we know mathematical objects exist by inference to the best explanation based on our best scientific theories. But that doesn't tell us what mathematical objects are *like*, and most importantly, it still doesn't answer the »makes no difference« argument. We may be forced to believe in them, but that doesn't tell us whether or not they actually *do* anything. I'd like to discuss one way that mathematical objects might *do* something. I think there is a useful and informative way we can talk about mathematical objects as causal. I do this by discussing a case of mathematical constraint as proposed by Marc Lange (2017). I will elaborate on the notion of mathematical constraint and talk about the constraint relation in general. I then discuss structural equation models and how they can be used to represent causal relationships. This fits particularly well with the interventionist view of causality that I will describe, and how it can be used as a test to determine which relationships are causal. I think that mathematical constraint passes this test. Using this framework, I will discuss a specific structural equation model that represents a constraint relationship. This specific constraint relationship is also clearly causal. I think that the structure of this relation naturally maps on to the structure of an archetypal example of a mathematical constraint. Not only do they share a common structure, but both relations behave in the same way in interventionist treatments. This should give us reason to say that mathematical constraint is causal. For those interested in the epistemology of mathematics, this may provide a workaround to Benacerraf-type objections; it *would* be puzzling how we know about acausal mathematical objects, but since mathematical objects are causal, we can explain our knowledge. For those who sympathize with or are concerned about the »makes no difference« argument, we again have an answer: mathematical objects are causal, so it would make a difference to the world if they did not exist.

Keywords: interventionism, counterfactual, structural equation model, causality, constraint.

⁶ An earlier version of this article appeared in *Dialectic*. Available at: <https://uoydialecticjournal.wordpress.com/>

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