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The nature of ψ : Epistemic and ontic views of the wave function

Narava ψ -ja: epistemični in ontični pogled na valovno funkcijo

Dilema o pomenu valovne funkcije je morda ena najbolj kontroverznih in spregledanih v sodobni fiziki. V znanstveni skupnosti obstaja široko nesoglasje glede tega, ali je treba na valovno funkcijo gledati kot na resnični fizični objekt ali pa ustreza stanju nepopolnega znanja o osnovnem sistemu, na primer gostoti verjetnosti. Ti pogledi se imenujejo ψ -ontični in ψ -epistemični. Cilj tega članka je dati jasen vpogled v problem in značilnosti obeh pogledov. Po nekaj razjasnitvenih uvodnih predstavah bom podala pregled, kako so to vprašanje zaznali in interpretirali pionirji kvantne teorije, kot sta Bohr in Einstein, nato pa nadaljujem z modernejšimi časi in imeni, kot so Bell, Kochen in Specker. Nato se bom osredotočila na sodobno literaturo in poskušala izčrpno predstaviti prednosti in slabosti vsakega pogleda. Na koncu bom razpravljala o nekaterih vprašanjih, ki nam jih postavlja vprašanje valovne funkcije, na primer o našem odnosu do resničnosti.

Ključne besede: kvantna mehanika, valovna funkcija, znanstveni realizem, ontologija.

1. Introduction

What *is* the wave function? We all know that it forms the core of the quantum mechanical formalism, that its dynamics are specified by the Schrodinger equation, and that Born's rule relates it to the probability of measuring a certain state or observable quantity. But should we regard them as something real, »part of the furniture of the world« (Callender, 2015), or rather as a representation of an incomplete state of knowledge about an underlying entity? The question of the ontological status of the wave function arises partly from wave-particle duality (Callender, 2015; Norsen, 2010), and partly from the measurement problem (Gao, 2017).

The wave function is the equation for »matter waves« proposed by the de Broglie hypothesis, but it lives in $3N$ -dimensional configuration space, while particles live in 3D space: are the latter emergent from the former, or are they two ontologically distinct entities? Is there a physical entity that corresponds to the wave, or is it

just a mathematical tool? What do entanglement and collapse in projective measurement tell us about the structure of reality? The wave function appears to be intrinsically different from the wave equations for electromagnetic radiation because of the space in which it propagates; this leaves open whether or not the wave behavior of matter should be considered as emanating from a physical entity such as a field.

These questions have been part of the foundations of quantum mechanics since its birth; they are essential for us to understand what does and does not belong to the ontology of the world. The theory seems to suggest the existence of an underlying physical reality (Gao, 2017), but what is it? And most importantly, what does the wave function mean in all this?

1.1 Mathematical formalism and physical entities

There are some caveats to be made when exploring this topic, as Maudlin points out in (Albert and Ney, 2013). We should keep in mind that the wave function itself is a mathematical object, let us call it ψ , and we should distinguish it from the quantum state, i.e., the entity it represents, λ . Our efforts to interpret ψ should not reify it and confuse it with λ , but try to investigate what is the relation between the two.

It is an open question whether the quantum state is uniquely determined by the wave function or whether there is something else in the theory, and whether the wave function directly and literally represents the quantum state or our state of knowledge about it. Does ψ supervene over the quantum state or not? The answers to these questions will determine our stance on the ontology and status of ψ . Unfortunately, the task of constructing an ontology starting from a theory so heavily laden with mathematical formalism complicates the distinction between what is part of the mathematical description and what is a physical entity, and exposes scientists to the risk of naively reading the metaphysics of a theory from its formal structure (Ney, 2015). (Norsen, 2010) notes that »mathematically equivalent formulations can have radically different implications in regard to ontology and causality«, and points to this risk. In recent years, however, the scientific community has embraced the idea that the question of the status of the wave function should be approached rigorously, using both mathematical and heuristic arguments, i.e. a thorough engagement with the formalism to find out what lies behind it (Leifer, 2014a).

Maudlin (Albert and Ney, 2013) paints a clear picture, albeit inevitably biased by his view, as Ney points out in (Ney, 2019), of what we should be looking for when determining the ontology of a theory. I think it is worth considering it as a kind of terminological clarification for what follows.

1. Primary observables: these are the phenomena, that is, what we directly observe as measurement outcomes. The choice of primary observables is itself a

theoretical act, and thus a stance, because they obviously do not correspond to the ontology of quantum mechanics. If we are realists, there are objective facts about the state of the world that are independent of observation; they may be different from phenomena, i.e. what we perceive about reality in measurements may not be a complete picture of it and we may not be able to grasp the underlying reality, but that underlying reality exists independently of our observations.

2. **Ontology:** these are the things we take to be physically real, which are supposed to explain the primary observations, following the laws. An epistemic distinction has been made between primary and secondary ontology: The primary observables supervene on the state of the primary ontology, but not on that of the secondary ontology, i.e. if we change the latter, the data remain the same, because the secondary ontology does not directly affect the behavior of the phenomena, but is rather theoretical in nature, whereas a change in the primary ontology affects a change in the observables. The secondary ontology is something that we postulate as real, but whose existence is not directly related to the outcome of our measurements. The wave function, if we consider it to be part of the ontology, will most likely be part of the secondary ontology, while other entities, such as particles, form the primary ontology. It is desirable that the elements that form the ontology of a theory are local, since we assume that the constituents of reality obey locality (Norsen, 2010). This follows from the fact that we require a theory to obey causality. Non-local causality would imply influence from a distance, thus generating non-determinism.
3. **Laws:** The behavior of primary and secondary ontology obeys dynamical laws. It is unclear whether we should conceive of laws as governing this behavior – this view is called *nomological primitivism* (Ney, 2019) – or rather as a mathematical description that scientists have devised to fit the spontaneous behavior of ontology (*Humean view*). Another interpretation of the status of laws is that they are spontaneous dispositions of ontology: In this case, they are not prescriptive, but they are related to properties of ontology (*dispositionalism*; see (Dorato, 2014; Ney, 2019; Dorato, 2015)).

1.2 Epistemic and ontic

The answers that physicists and philosophers have given to these questions initially suggest two views: ψ complete views and theories of hidden variables. The first view assumes that the wave function is »all there is«, i.e. the only element of our ontology (Goldstein, 2011). The best known interpretation of this kind is the Many-Worlds or Everettian view.

In this review, however, I will focus on the second view, according to which the ontology is not exhausted in the wave function, but is complemented by other

elements, mainly due to the results of the 1935 EPR paper (Einstein, Podolsky and Rosen, 1935), discussed in section 2.2. Within this framework, the wave function can be considered as part of the ontology or not; in other words, it can be considered as indicating a real physical entity or as a simple tool that allows us to derive some information about the underlying reality (Harrigan and Spekkens, 2010).

The first view is called ψ -ontic, the second ψ -epistemic. As (Harrigan and Spekkens, 2010) put it, »In ψ -ontic models, distinct quantum states correspond to disjoint probability distributions over the space of ontic states, whereas in ψ -epistemic models, there exist distinct quantum states that correspond to overlapping probability distributions«. An epistemic model allows a quantum state, λ_x , to be in support of two distinct probability distributions that overlap in the region occupied by it, $|\psi_\alpha\rangle$ and $|\psi_\beta\rangle$. In contrast, there is no such state in an ontic model: disjoint probability distributions correspond to different ontic states.

Formally, a hidden-variable theory is ψ -ontic if, for arbitrary preparation procedures P_ϕ, P_ψ , associated with different wave functions ϕ and ψ , the probabilities of obtaining a quantum state λ ,

$$p(\lambda | P_\phi)p(\lambda | P_\psi) = 0$$

and epistemic otherwise (Harrigan and Spekkens, 2010). This means that in an ontic model the quantum state supervenes on the wave function, while in an epistemic model this is not the case.

Moreover, we can distinguish between realist and antirealist epistemic views: The former claim that although there is an underlying ψ , the wave function does not describe it directly, but rather our state of knowledge about it, which is considered incomplete and probabilistic. The latter, on the other hand, states that we cannot assume that there is a ψ -state that is different from ψ , blurring the distinction between the state and the wave function. This article will focus more on the realist views than on the antirealist or instrumentalist views.

2. Historical overview

In the last chapter I outlined the problem at hand and clarified the distinction between epistemic and ontic views of the wave function. Now it is worth looking at the debate from a historical perspective, since the question of the nature of the wave function has long been overlooked. This was probably because the orthodox Copenhagen interpretation was fundamentally instrumentalist and antirealist about ψ , i.e. a »shut up and calculate!« attitude – to quote David Mermin – prevailed (Dorato, 2015). The wave function was perceived as epistemic and subjective, only as an operational tool, and its meaning was not explored. However, this

canonical picture is partly due to an oversimplification of Bohr's views (Halvorson, 2019; Ladyman, 2019), and since Einstein there have been attempts to investigate the topic further, to find limitations and difficulties for epistemic models, and to make new ontic proposals.

2.1 Bohr and the Copenhagen interpretation

The Copenhagen interpretation, led in particular by Bohr et al¹, is often simplistically presented as alternatively antirealistic, operationalist, or instrumentalist. It is commonly seen as an interpretation of the wave function as an accounting device, a computational tool representing probabilistic information about the results of measurements, according to Born's rule. Although this is consistent with some of Bohr's views, the popular picture has retained only its most extreme features.

(Bohr, 1948) highlights the difficulty of assigning physical properties to the states represented by the quantum mechanical formalism; for this reason he gives ψ a symbolic interpretation and uses it exclusively as a tool for deriving predictions. The motivation for this is not an antirealist or skeptical view of physical reality, but derives from the highly abstract nature of the wave function, which makes it difficult to reconduce it to any physical concept.

As noted in particular in (Ladyman, 2019), Bohr explicitly acknowledged the existence of particles, e.g. in his Nobel Prize lecture, and therefore cannot be seen as rejecting the idea of an objective, observer-independent physical reality. However, the quantum mechanical formalism is so different from our classical perception that it would be risky to interpret it realistically: We can only observe phenomena and make probabilistic predictions about them.

Therefore, the concept of causality, which is crucial for scientific explanation, has to be replaced by the concept of complementarity, i.e. by attributing classical features to quantum description, in order to try to cope with the fact that we are unable to deal with quantum mechanical concepts because of the way science and our perception of the world have developed. As Bohr puts it, this problem of interpretation has nothing to do with metaphysics, but rather with language: our language as scientists is so biased with classical notions that it cannot picture ontology at the quantum level and therefore must use the concept of complementarity to try to make sense of it.

In any case, we are denied a full understanding or even observation of what is real at the quantum mechanical level, and the wave function is just a tool cast in a language that mimics the classical wave equation, allowing us to derive predictions. As (MacKinnon, 2012) summarizes, the Bohr interpretation is minimal rather than antirealistic, sufficient to make predictions about experiments without in-

¹ Described in MacKinnon, 2012.

consistency, and thus it holds that the wave function is an epistemically complete description of reality. ψ is then not part of the ontology, but knowledge of the ontology is closed to us, so the wave function provides us with all the information we need and care about in the system.

2.2 Einstein and the EPR paper

Einstein was a staunch critic of Copenhagen Quantum Mechanics, as he hoped for a new theory that could restore realism and determinism and that would make the probabilistic treatment of ψ only a matter of epistemic imprecision rather than an intrinsic limit. In other words, he hoped to establish a realist ontology given over to the minimal interpretation of Bohr. He was convinced that the wave function was an ontologically incomplete description, since it could only describe ensembles of systems (Fine, 1993).

Fine points out that Einstein's position on quantum mechanics was never very clearly stated and that we should take into account that his view is skewed by the desire to push the scientific community towards the new theory mentioned above. However, we can characterize his view as instrumentalist, subjective, and a proponent of hidden variables. He holds that ψ is an ensemble description because it describes only the statistics for measurement outcomes, according to Born's rule: it is thus subjective because it describes only our knowledge of the state of the system, and collapse corresponds to information updating. Thus, his epistemic view of the nature of the wave function must be considered negative.

His proposal for a model with hidden variables, together with an argument for the incompleteness of ψ , can be seen in the EPR paper of 1935 (Einstein, Podolsky and Rosen, 1935). Even though the paper was written almost entirely by Podolsky and somewhat pushes and modifies Einstein's views, as noted in (Harrigan and Spekkens, 2010; Fine, 1993), I will still consider it as the first example of a formal proof to investigate the meaning of the wave function.

The EPR paper argues for the ontological incompleteness of ψ by invoking a kind of »reality criterion« (Fine, 1993 which states that »If, without in any way disturbing a system, we can predict with certainty [...] the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity« (Einstein, Podolsky and Rosen, 1935).

It follows that if we consider observables described by two non-commuting operators, either the wave function description is not complete, or two non-commuting quantities cannot have simultaneous reality. To show that the first possibility holds, the paper considers two systems that are separated after a certain time of interaction; the state of one system cannot be determined exactly because the two wave functions are superimposed. Different measurements can be made on the first system: Thus, the wave function of the combined system can be expressed in different terms.

$$\Psi(x_1, x_2) = \sum_{n=1}^{\infty} (\psi_n(x_1)u_n(x_2))$$

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The paper then states that »As a consequence of two different measurements performed upon the first system, the second system may be left in states with two different wave functions«, although no objective change can occur on the second system since the systems no longer interact. This is just to establish the many-to-one correspondence of wave functions to reality. The two states that the second system then still has can be eigenfunctions of non-commuting operators. When we make a measurement on the second system, we can only determine one of the two eigenvalues at a time. However, when measuring on the first system, both quantities seem to be reality at the same time. Therefore, the description by the state function cannot be complete.

The paper then explicitly calls for theories of hidden variables as the only way to give the wave function a realistic ontological status. Therefore, the wave description of matter alone is insufficient to provide a complete prediction of reality.

2.3 No-go theorems

After the EPR paper, many other papers appeared in the context of hidden variable models that attempted to determine the meaning of the wave function in a formal way. These use quantum mechanical toy models, called *ontological models*, to derive necessary results about the state of ψ using formal and rigorous mathematical arguments, following the method used in the EPR paper (Leifer, 2014a). As explained by (Leifer, 2014b), ontological models are »attempts at explaining quantum predictions in terms of real physical properties (λ -ontic states) existing independently of the experimenter«.

The first notable no-go theorem for hidden variables is Bell's theorem (Bell, 1966), which challenges the refutation of the existence of hidden variables in quantum mechanics by Von Neumann. Bell goes further than EPR and highlights the fact that hidden variables, even if they exist, cannot appear in a simple measurement, as this would render quantum mechanics observationally inadequate, thus emphasizing the sound but incomplete character of the state function description. For Bell, »The question at issue is whether the quantum mechanical states can be regarded as ensembles of states further specified by additional variables, such that given values of these variables together with the state vector determine precisely the results of individual measurements« (Bell, 1966). The paper refutes the assumptions in the proof of Von Neumann and thus establishes the necessary existence of hidden variables and, most importantly, shows that models

with hidden variables represent non-local and non-separable trajectories, which resolves the EPR paradox by showing a spacelike interaction between systems and decisively challenges the concept of space-time and causality in quantum mechanics. Systems are thus instantaneously affected by distant systems, producing an undesirable non-local causality. The results of Bell's work can then be summarized by a paradox: Either hidden variables must exist, and therefore ψ must be supplemented by an additional ontology, but the wave function is shown to behave nonlocally, or the wave function is complete, which is not the case by the EPR work, but local.

As (Harrigan and Spekkens, 2010) show, Bell's theorem is only needed for ontic models with hidden variables to rule out locality, since epistemic models are intrinsically nonlocal. In other words, as noted by (Norsen, 2010), for any theory in which the wave function has beable status, it is necessarily a non-local beable.

So far, then, the no-go theorems suggest that the most accurate interpretation of the wave function should be a ψ -incomplete ontic view in which the ontology is instantiated by some hidden variables that could be local beables and a non-locally behaving wave function that has real physical existence in some way. We will see that hidden variables and nonlocality are not the only constraints.

The 1967 Kochen-Specker paper, which deals with the proof of the existence of hidden variables in quantum mechanics, shows, by providing an epistemic ontological model in 2 dimensions, that for any Hilbert space with a number of dimensions $d \geq 3$, deterministic ψ -epistemic theories are necessarily measurement contextual (Kochen and Specker, 1967). (Aaronson et al. 2013) further show that epistemic theories in 3-dimensional Hilbert space and above cannot satisfy symmetry under unitary transformations and maximal nontriviality, i.e., overlap of probability distributions over non-orthogonal states.

Finally, (Pusey, Barrett, and Rudolph, 2012) shows the inconsistency of nontrivial ψ -epistemic theories: If we assume a separability condition, the wave function must be ontic.

Thus, it seems that all no-go theorems point in the direction of a ψ -ontic theory of hidden variables, which may be mathematically consistent and correctly reproduce the quantum mechanical predictions, restricting the possibility of finding non-trivial and accurate ψ -epistemic models. However, we will see in the next section that there are problems with the ontic views that are best solved by adopting an epistemic perspective. The formal strategy of no-go theorems, though it has certainly shed some light, is far from resolving the question of the ontological status of the wave function.

3. The epistemic-ontic debate

3.1 Configuration space realism

According to the so-called ontic views of the wave function, ψ constitutes a real physical entity that is part of the ontology, even if it is not directly observable. Thus, when we talk about it, we refer to a function that describes a kind of field, just as the electromagnetic wave equation describes the electromagnetic field. This is exactly the view held by the supporters of wave function realism (Albert and Ney, 2013; Gao, 2017; Ney, 2015, 2019).

Wave function realism takes ψ to represent a field in configuration space, i.e., the multidimensional space containing all possible configurations of any system in the universe, modeled by a Hilbert vector space with potentially infinitely many dimensions. In fact, this is the space in which the wave function »lives in« as a mathematical object: wave function realism assumes that this space has a literal corresponding physical counterpart, and is not purely abstract.²

This configuration space is then conceptually very different from the phase space we encounter in thermodynamics and statistical mechanics as the space of all possible microstates, because it does not represent a space of probabilities or an ensemble state space, but a concrete alternative to our familiar 4-dimensional spacetime, as (Ney, 2015) suggests. Configuration space, as the space in which the wave function operates, would then be the fundamental physical space of the world, while 3-dimensional space or 4-dimensional spacetime would be contingently emergent. Albert (Albert and Ney, 2013) explains the generation of geometric appearances as a matter of dynamics: an adequate coordinate transformation in n -dimensional space would generate the apparent 3-dimensional one. In other words, while it may appear to us that we live in a 4-dimensional manifold of 3-D space and time, the real structure of »space« is that of a multidimensional one, over which there is a field called » ψ « that describes the behavior of matter.

However, if we consider quantum field theory, the configuration space itself is defined from the field operators, which in turn have a connection to spacetime, so it may just be a matter of convention which one we consider fundamental (Myrvold, 2015).

The wave function realist view is compatible with both a ψ -complete and a hidden variable ontology: in the former case, the wave function would determine the emergence of particles and all our observables; in the latter case, it would be paired with other local beables for which we can only partially determine properties (Albert and Ney, 2013; Ney, 2015).

² See (Halvorson, 2019) on literal interpretation on the quantum state.

Configuration space realism would provide an explanation for the phenomenon of entanglement and restore locality; as pointed out by (Norsen et al., 2015), quantum phenomena such as the violation of the Bell inequality and quantum teleportation, for example, would require configuration space realism.

Following (Ney, 2015), suppose we have a system with two electrons that are in an indeterminate x -spin state. Their state before the measurement can be represented in two ways:

$$\psi_{singlet} = \frac{1}{\sqrt{2}} |x_{\uparrow}\rangle_1 |x_{\downarrow}\rangle_2 - \frac{1}{\sqrt{2}} |x_{\downarrow}\rangle_1 |x_{\uparrow}\rangle_2$$

$$\psi_{independent} = \frac{1}{2} |x_{\uparrow}\rangle_1 |x_{\uparrow}\rangle_2 + \frac{1}{2} |x_{\uparrow}\rangle_1 |x_{\downarrow}\rangle_2 + \frac{1}{2} |x_{\downarrow}\rangle_1 |x_{\uparrow}\rangle_2 + \frac{1}{2} |x_{\downarrow}\rangle_1 |x_{\downarrow}\rangle_2$$

If we send the electrons in opposite directions to two Stern-Gerlach devices and deflect them to detect their x -spin, we assume that the first electron has 2 possible positions, A and B , while the second one can be in positions C or D . A or C results when the electrons are in a spin-up state, B or D when they are in a spin-down state. The state of the system can be represented as:

$$\psi_1 = \frac{1}{\sqrt{2}} |A\rangle_1 |D\rangle_2 - \frac{1}{\sqrt{2}} |B\rangle_1 |C\rangle_2$$

$$\psi_2 = \frac{1}{2} |A\rangle_1 |D\rangle_2 + \frac{1}{2} |A\rangle_1 |C\rangle_2 + \frac{1}{2} |B\rangle_1 |C\rangle_2 + \frac{1}{2} |B\rangle_1 |D\rangle_2$$

The wave function realism allows us to draw a distinction between these two states, since each combination occupies a different point in the configuration space. Then the wave function will have different locations in the two cases, while if we consider only the probability, we will get the same information about the system in both cases. Then this interpretation does not take the trajectory of the system as non-locally distributed between different possibilities, because the wave function results local in the configuration space. The configuration space allows us to satisfy the separability requirement, i.e., that the physical state supervene on the specification of the local beables at any point in spacetime (Myrvold, 2015).

By making the configuration space physical and fundamental, we would solve the paradoxes raised by the no-go theorems. However, we should point out that although locality is recovered in configuration space, the beables in 3-D space remain nonlocal (Norsen, 2015). This raises the problem of the interaction between

high-dimensional wave function and low-dimensional peaks, i.e., particle-like beables, which will be further explored in the next section.

(Gao, 2014, 2017) solves this problem while maintaining the fundamental character of our familiar spacetime by explaining entangled states described by the wave function with random discontinuous motion of particles. He also proposes a particle ontology by stating that since the wave function of an N-body system corresponding to N particles has $3N$ coordinates in configuration space, these must correspond to 3-dimensional positions.

3.2 Problems with ontic views

As stated by Heisenberg in 1955, cited in (Bacciagaluppi and Valentini, 2006),

»For [de Broglie and] Bohm, particles are 'objectively real' structures, like the point masses of classical mechanics. The waves in configuration space are also objectively real fields, like electric fields... [But] what does it mean to call waves in configuration space 'real'? This space is a very abstract space. The word 'real' goes back to the Latin word 'res' meaning 'thing'; but things are in ordinary three-dimensional space, not in an abstract configuration space«.

By restoring locality and separability, configuration space realism offers a viable solution to the problems posed by no-go theorems about hidden variable theories. However, it has significant problems that could be arguments for an epistemic view (Gao, 2017).

The first and most obvious is how to recover the 3-dimensional space of our experience and observations from a multidimensional ontology. As pointed out by Maudlin in (Albert and Ney, 2013), the observables should be directly determined by the ontology: The wave function should then be able to specify from the configuration space for each N-body system the positions of the particles that we detect in 3-D space. However, there are many ways in which N particles can move in 3-D space, all compatible with only one evolution in $3N$ -D space, and nowhere can we find a specification within ψ of which dimensions correspond to which particles (Monton, 2002). There is no »natural« correspondence in the Hamiltonian that specifies how objects in $3N$ -D space correspond to three-dimensional ones, and we cannot even find one in some physical facts. (Myrvold, 2015) claims that the problem is specifically finding 3-D objects in the wave function, not in the relation of the wave function to ordinary space. This is because in this picture the fundamental space of the world is configuration space, therefore the wave function would have virtually no relationship to it. The problem of emergence arises when we need to identify a correspondence between object positions in configuration space and in 3-D space, and it is not an obvious one, since such a correspondence would be required to preserve locality and separability, so the physical state supervene on a specification of local conditions at each point. Note that the

physical state in 3-D does not supervene on the state of the wave function (Mon-ton, 2002).

The second problem with ontic views is how to treat the collapse of the wave function in measurement (Gao, 2017; Ney, 2015). According to an epistemic view that does not treat the wave function as a representative of something real, the collapse corresponds merely to an update of the information we have about a system. In contrast, we must take collapse seriously if we consider the wave function as a direct representation of a state. It then becomes an undefined but real physical process that instantaneously changes the wave function and thus the state, which in turn introduces indeterminism and non-locality into the description of physical processes. Indeed, as proved by (Harrigan and Spekkens, 2010), locality is also ruled out for ontic views, and this is far from unproblematic. If collapse is a physical process that cannot be explained, the only ψ ontic theory that could avoid this problem is the Many Worlds interpretation, but believing in an infinite branching of reality is, after Occam's razor, less likely to get us to the truth than postulating that collapse is either an epistemic or a nonlocal physical process (although allowing for nonlocality is still pretty bold). In a configuration space realist view, collapse could be viewed as the movement of the wave function to a precise position in configuration space, but this too is instantaneous.

The third problem is the indistinguishability of nonorthogonal states (Leifer, 2014b). From an ontic point of view, two nonorthogonal states represent different arrangements of physical reality, but the distinction cannot be detected in any way. This could in principle be explained by an epistemic view according to which, since nonorthogonal quantum states correspond to overlapping probability measures about the true ontic states, indistinguishability results from the fact that the ontic state is in the overlap region. However, Leifer shows that even this epistemic explanation becomes imprecise and implausible for higher Hilbert space dimensions.

The last point is the eigenvalue-eigenstate half link, i.e., the fact that »when a physical system is in an eigenstate of an observable, the system has an observation-independent property, with value being the eigenvalue corresponding to the eigenstate« (Gao, 2017). The wave function of a system, assuming that it corresponds to something real, would then be determined by the set of observables of which it is an eigenstate. But is this only the case if the wave function is an eigenstate of some observables? In other words, does the quantum state have properties only if it is specified by the wave function as eigenvalues of observables? If the wave function is real, then shouldn't the properties always be present, and if that is the case, why aren't they always detectable? This is related to the crucial fact that there is a many-to-one correspondence of wave functions to quantum states, which seems to undermine the onticity of the wave function.

All of these problems relate to a configuration space realist view as we consider it the main proponent of ontic models; if we were to consider other views according to which the wave function is real but not in configuration space, entanglement would become a physical process that could not be explained, hence such views would be far more problematic than configuration space realism (Harrigan and Spekkens, 2010; Ney, 2019). Therefore, I will not consider them as viable options.

The problems we have just considered can be solved by an epistemic view; however, we must note that the concept of epistemic view is very broad and can account for various degrees of antirealism (Gao, 2017). The exact formulation of our wave function ontology depends on the preferred version of QM and is therefore a stance rather than a discovery, allowing us to interpret the theory differently (Monton, 2002). Certainly, to arrive at a more accurate ontology, we need to consider different formulations when the only source from which we can derive it is a mathematical formalism (Albert and Ney, 2013; Callender, 2015). As we have already seen, ψ -epistemic views are strongly undermined by no-go theorems that prove their limited nature. However, they might solve some of the difficulties raised by ontic views. I think epistemic views are worth considering as operational theories that leave aside the question of the status of the wave function and adopt a classical particle ontology, treating the wave function in its abstract and nonlocal nature as a mathematical tool that is useful but mysterious. For example, (Bartlett et al., 2012) have attempted to develop an epistemic version of quantum mechanics, Gaussian Quantum Mechanics, by implementing classical Liouville mechanics with the uncertainty principle, but, as the authors themselves point out, the model is limited and is proposed as an exercise in axiomatization.

In the final section, I will explore the philosophical implications of our answers in relation to the ontological status of the wave function, focusing on the notion of law, mathematical object and causality, and how formalism and ontology can be related.

4. Interpretive perspectives

It seems, then, that reducing the debate about the ontological status of the wave function to its core raises the question of what the fundamental physical space of reality is and what exists in that space. Configurational space realism presents us with what (Solé, 2013) calls the problem of perception and the problem of missing invariants; its main difficulty is explaining the 3-dimensional character of our experience, which should be determined in some way by ontology, as indicated in chapter 1, and the fact that the dynamical symmetries of non-relativistic quantum mechanics also seem to suggest a 3-D space. The emergence of 3-D beables is then still not accounted for. In addition, a mixed ontology with both

high-dimensional ψ - and low-dimensional particles presents us with the problem of communication, i.e., how to establish a connection that causally explains the influence of the former part of the ontology on the latter.

Since ontic views face these obstacles and realistic epistemic views are ruled out by no-go theorems, it is questionable whether there really is a well-defined and knowable quantum state λ , since its relation to ψ and to local beables is puzzling for both epistemological and ontological reasons. We are then tempted to think of λ as related to the Kantian noumenon: the state that exemplifies reality but is intrinsically impossible to know, and unlike the phenomenon that is our only possible epistemic object (Kant, 1934).

Thus, as (Callender, 2015) suggests, the problem can be viewed in part as a two-space problem, borrowing a Cartesian notion: How can we connect the epistemically inaccessible fundamental space of reality – the space in which λ exists – with the space we map into our knowledge with models and theories, the one in which we construct our laws and ontology? Which is the 3-D space and which is the configuration space? Can we consider our observations and measurements as part of epistemic space or ontic space? Although this is a very broad question and I cannot possibly explore the answer in this review, I will focus on the views expressed in the literature. The main solutions to this »separation in spaces« have been of opposite natures: either enlarging the ontic space, i.e. the space where particles arguably live, into configuration space, as Albert suggests, or, following (Norsen, 2010), reducing the space where the wave function lives to a 3D space.

Rather than putting the two parts of our ontology in the same space, according to (Callender, 2015), the problem of communication can also be solved by eliminating part of the ontology: either we can consider local beables as emerging from the wave function, or we can eliminate the wave function in favor of a particle ontology and treat ψ as abstract.

This seems pretty much an interpretive stance: How can we know exactly whether the wave function is ontological or merely operational? Why do we tend to reify the wave function by associating it with a direct representation of the quantum state of reality, even though it cannot fully describe ontology? (Callender, 2015) suggests that we look at the mathematical formalism to solve this puzzle. We have no problem accepting that the Hamiltonian function in the Hamilton-Jacobi formulation of classical mechanics lives in multidimensional phase space because we consider it a mathematical object and a lawlike entity: Why can't we consider the wave function as related to the Hamiltonian function (Norsen, 2015)? Considering ψ as a lawlike entity would resolve the debate between epistemic and ontological status by definitively removing ψ from ontology, although its role in determining quantum state remains.

However, this shifts the same debate about the nature of a law: are the laws of physics primitive, i.e. they generate and govern the world, or dispositional or

Humean, in that they arise as descriptions of intrinsic tendencies of matter (Callender, 2015; Dorato, 2015)? The notion of a nomological entity is also unclear and should be treated with caution (Ney, 2019; Dorato and Laudisa, 2015). Ney criticizes the primitivist view of laws, but there still seems to be a sense in which the laws of nature seem to be causative and not merely descriptive. This would be the case of ψ if we view it as nomological, since it does not supervene on particle positions.

Indeed, laws are not necessarily tied to one mathematical formalism: Classical mechanics can be cast in a quantum Koopman-Von Neumann framework, and quantum mechanics can be cast in a classical Hamilton-Jacobi formalism (Callender, 2015). The way laws are formulated depends only on the set paradigm and can therefore be considered internalistic (Kuhn, 1996). However, the quantum wave function differs from the classical wave function in that it is linear and single-valued and acts as a causal agent that generates the motion of objects (Callender, 2015).

This fact raises some difficulties in considering ψ as nomological (Goldstein, 2011; Dorato and Laudisa, 2014): laws are not supposed to be dynamical entities, i.e. to have explicit time dependence; moreover, ψ is contingent and can be prepared as an initial condition: May we consider a law of nature as contingent? Moreover, the many-to-one correspondence between wave function and quantum state seems to reject the nomological nature of ψ^3 (Ney, 2019). For these reasons, (Dorato and Laudisa, 2014) argue that nomological realism is inconsistent because if we consider the wave function as a mathematical object, as a law is supposed to be, it must be interpreted instrumentally or minimally, and not as fundamentally causally active. In other words, a nomological view of the wave function can only require an abstract ontology, since the mathematical nature of laws implies that they cannot be primitive, but only Humean. Therefore, a nomological view can only be instrumental and laws cannot govern beables without being instantiated as physical entities.

If we consider the wave function as part of the laws of nature, then it is still unclear whether it is a description of the evolution of a system or the information we have about it, or whether it governs it as another entity, for example, the equivalent of a field. If the latter is the case, there would have to be an entity in spacetime equivalent to the wave function that governs the beables. So we are left with our original question, and as in Bohr's day, the safest answer is to suspend our judgment about the wave function and treat it only operationally.

³ Note that Bohmian mechanics proponents, such as (Norsen, 2015), tend to consider the universal wave function, Ψ , instead of the conditional wave function, ψ , as lawlike. However, even Ψ is a solution to a dynamical equation, namely the Wheeler-deWitt equation, and it is dubious how a law of nature can be the solution to an equation of motion.

5. Conclusion

Finally, I would like to make a brief remark about what it means to be a realist with respect to the wave function. We have seen that, no matter how formal may be our methods, no mathematical proof has yet been able to clarify the ontological status of the wave function; so it seems that realism with respect to it must be an interpretive stance. What then does it mean to judge the best interpretation of the status of the wave function? Are we looking for empirical coherence and empirical accuracy, or rather metaphysical truthfulness? Will there ever be a way to know with certainty what the wave function is and how it relates to the quantum state when our observations seem inherently incomplete? Would we have to postulate a partial reality in which properties are instantiated only when the corresponding operators commute in measurement? In other words: Is realism about the wave function and the quantum state even desirable, or should we settle for a phenomenological metaphysics? As (Dorato and Laudisa, 2014) argue, we can be realists about quantum mechanics, the quantum state, and therefore about the reality of the world we see, even if we choose not to be realists about the wave function. What is certain is that a realist attitude will be beneficial in the search for the truth about ψ (Ladyman, 2019; Halvorson, 2019). For now, the assumption that »there is an underlying reality« is surely the most comforting to uphold (Gao, 2017): no matter how much quantum mechanics hints at skeptical scenarios, the existence of reality will always be a cornerstone of our beliefs.

The nature of ψ : Epistemic and ontic views of the wave function

The dilemma concerning the meaning of the wave function is perhaps one of the most controversial and overlooked in contemporary physics. There is widespread disagreement in the scientific community as to whether the wave function should be viewed as a representative of a real physical object or whether it corresponds to a state of incomplete knowledge about the underlying system, such as a probability density. These views are respectively called ψ -ontic and ψ -epistemic. This article aims to give a clear insight into the problem and features of both views. After some clarifying preliminaries, I will give an overview of how the question was perceived and interpreted by the pioneers of quantum theory, such as Bohr and Einstein, and then move on to more modern days with the likes of Bell, Kochen, and Specker. Then I will focus on the contemporary literature and try to give a comprehensive account of the pros and cons of each view. Finally, I will discuss some of the questions that the question of the wave function raises for us, such as our relationship to reality.

Keywords: quantum mechanics, wave function, scientific realism, ontology.

References

- Aaronson, S. et al. (2013). »Psi-Epistemic Theories: The Role of Symmetry». arXiv.org 88.3.
- Albert, D. and Ney, A. (2013). *The wave function: Essays on the metaphysics of quantum mechanics*. New York; Oxford: Oxford University Press.
- Bacciagaluppi, G. and Valentini, A. (2006). »Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference«.
- Bartlett, S., Rudolph, T., and Spekkens, R. (2012). »Reconstruction of Gaussian quantum mechanics from Liouville mechanics with an epistemic restriction«. arXiv.org 86.1.
- Bell, J. S. (1966). »On the Problem of Hidden Variables in Quantum Mechanics«. *Reviews of Modern Physics* 38.3, pp. 447–452.
- Bohr, N. (1948). »On The Notions of Causality and Complementarity«. *Dialectica* 2.3-4, pp. 312–319.
- Callender, C. (2015). »One world, one beable«. *Synthese* 192.10, pp. 3153–3177.
- Dorato, M. (2015). »Laws of nature and the reality of the wave function«. *Synthese* 192.10, pp. 3179–3201.
- Dorato, M. and Laudisa, F. (2014). »Realism and instrumentalism about the wave function. How should we choose?«.
- Einstein A., Podolsky B., and Rosen N. (1935). »Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?« *Physical Review* 47.10, pp. 777–780.
- Fine, A. (1993). »Einstein's Interpretations of the Quantum Theory«. *Science in Context* 6.1, pp. 257–273.
- Gao, S. (2017). *The Meaning of the Wave Function: In Search of the Ontology of Quantum Mechanics*. Cambridge: Cambridge University Press.
- Gao, S. (2014). »The Wave Function and Particle Ontology«.
- Goldstein, S. and Zanghi, N. (2011). »Reality and the Role of the Wavefunction in Quantum Theory«. arXiv: 1101.4575.
- Halvorson, H. (2019). »To Be a Realist about Quantum Theory«. In Olimpia Lombardi et al., ed., *Quantum Worlds: Perspectives on the Ontology of Quantum Mechanics*. Cambridge University Press, pp. 133– 63.
- Harrigan, N. and Spekkens, R. (2010). »Einstein, Incompleteness, and the Epistemic View of Quantum States«. *Foundations of Physics* 40.2, pp. 125–157.
- Kant, I. (1934). *Critique of pure reason*. Abridged ed. London: Macmillan.

- Kochen, S. and Specker, E. (1967). »The Problem of Hidden Variables in Quantum Mechanics«. *Indiana University Mathematics Journal* 17.1, pp. 59–87.
- Kuhn, T. S. (1996). *The structure of scientific revolutions*. 3. ed.. Chicago: University of Chicago Press.
- Ladyman, J. (2019). »What Is the Quantum Face of Realism?« In Olimpia Lombardi et al., ed., *Quantum Worlds: Perspectives on the Ontology of Quantum Mechanics*. Cambridge University Press, pp. 121–132.
- Leifer, M. (2014a). »Is the quantum state real? An extended review of ψ -ontology theorems«. arXiv.org 3.1.
- Leifer, M. (2014b). » ψ -Epistemic Models are Exponentially Bad at Explaining the Distinguishability of Quantum States«. *Physical Review Letters* 112.16.
- MacKinnon, E. (2012). »Interpreting Physics Language and the Classical/Quantum Divide«. *Boston Studies in the Philosophy of Science*, 289. Dordrecht: Springer Netherlands.
- Monton, B. (2002). »Wave Function Ontology«. *Synthese* 130.2, pp. 265–277.
- Myrvold, W. (2015). »What is a wavefunction?« *Synthese* 192.10, pp. 3247–3274.
- Ney, A. (2015). »Fundamental physical ontologies and the constraint of empirical coherence: a defense of wave function realism«. *Synthese* 192.10, pp. 3105–3124.
- Ney, A. (2019). »Locality and Wave Function Realism«. In Olimpia Lombardi et al., ed., *Quantum Worlds: Perspectives on the Ontology of Quantum Mechanics*. Cambridge University Press, pp. 164–182.
- Norsen, T. (2010). »The Theory of (Exclusively) Local Beables«. *Foundations of Physics* 40.12, pp. 1858–1884.
- Norsen, T., Marian, D., and Oriols, X. (2015). »Can the wave function in configuration space be replaced by single-particle wave functions in physical space?« *Synthese* 192.10, pp. 3125–3151.
- Pusey, M., Barrett, J., and Rudolph, T. (2012). »On the reality of the quantum state«. *Nature Physics* 8.6, pp. 475.
- Solé, A. (2013). »Bohmian mechanics without wave function ontology«. *Studies in History and Philosophy of Modern Physics* 44.4, pp. 365–378.