

WHAT ONE CAN KNOW: FITCH'S ARGUMENT AND ITS CONSEQUENCES

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Abstract The paper, motivated by the chapter in Šuster's book, considers the aspect of the so-called Fitch's argument (FA) that seriously challenges the verificationist theory. Contrary to Šuster's view, it is throughout the paper that I'm pursuing the idea that most of the attempts that intend to vindicate verificationism from the grip of Fitch's argument, including Edgington's theory, fail in their intention. Concerning the attempts to mitigate the effect of Fitch's argument to verificationism in the framework of classical logic (Edgington as the most important representative), I'm siding with their critics (Williamson, Percival) and claim that they fail in their intention. Regarding the attempts to block the effect of Fitch's argument in the framework of non-classical (intuitionistic, relevant, dialetheist, and so on) logic, they do it by introducing principles that invalidate some of the basic classical rules and principles, usually introducing trivial worlds. In that case, the verificationist principle (as well as all inferences included in Fitch's argument) is vacuously valid, which seems to be unsatisfactory. In any case, there is no decisive evidence that either classical or any of the non-classical approaches can avail the verificationist anything to escape out of the grip of FA.

Keywordsverificationism,
anti-realism,
Fitch's paradox,
classical logic,
non-classical logic

**KAJ LAHKO VEMO: FITCHEV ARGUMENT IN
NJEGOVE POSLEDICE**

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Izvleček Članek, ki je motiviran na osnovi poglavja v Šusterjevi knjigi, obravnava vidik tako imenovanega Fitchevega argumenta (FA), ki predstavlja resen izziv za verifikacijsko teorijo. V nasprotju s Šusterjevim stališčem v celotnem prispevku zasledujem stališče, da večina poskusov, ki nameravajo verifikacionizem ubraniti izpod primeža Fitchevega argumenta, vključno z Edgingtonovo teorijo, ne uspe. Kar zadeva poskuse omilitve posledic Fitchovega argumenta za verifikacionizem v okviru klasične logike (Eddington kot najpomembnejši predstavnik), se postavljam na stran njihovih kritikov (Williamson, Percival) in trdim, da jim njihova namera ni uspela. Kar zadeva poskuse blokiranja vpliva Fitchevega argumenta v okviru neklasičnih (intuicionističnih, relevantnih, dialetheističnih in tako naprej) logik, to počnejo z uvajanjem načel, ki razveljavljajo nekatera osnovna klasična pravila in principe, pri čemer običajno uvajajo trivialne svetove. V tem primeru je verifikacionistično načelo (kot tudi vsi sklepi, vključeni v Fitchev argument) prazno resnično, kar ni zadovoljivo. V vsakem primeru pa ni prepričljivih dokazov, da bi bodisi klasični bodisi kateri koli od neklasičnih pristopov verifikacionistu lahko pomagal, da bi se rešil iz primeža FA.

Ključne besede
verifikacizem,
antirealizem,
Fitchev paradoks,
klasična logika,
neklasična logika

1 Introduction

In his excellent book *Modal Catapults* (Šuster, 2023), which, regarding the modal logic, I consider to be the most valuable publication in the area, Danilo Šuster devotes considerable attention to *Fitch's argument* (Fitch, 1963, also called *Church-Fitch's paradox*). The importance of Fitch's argument (FA) lies in the fact that it seriously challenges the verificationist theory, a distinguished form of anti-realism. Fitch's argument is, therefore, at the very centre of the fierce discussion between verificationists and their opponents, the realists. Though Šuster's book addresses many other interesting topics, this paper focuses only on this issue. Šuster provides an elegant and clear presentation of the argument and, when providing the interpretation of attempts to make the argument less harmful for the verificationist position, he focuses on Edgington's proposal (Edgington, 1985). Šuster himself (Šuster, 2023, 92) seems to be inclined to attempts that intend to rescue verificationism from the grip of Fitch's argument, particularly Edgington's solution. He holds (Šuster, 2023, 92) that "it [Edgington's solution] is the closest and most in the spirit of the semantics of modal logic". I agree with this qualification, and I also accept that among various attempts tending to save verificationism, Edgington's solution is "most in the spirit of semantics of modal logic" in the framework of classical logic. However, many other attempts to vindicate verificationism are trying to find their way into the framework of non-classical, non-standard logic. Nonetheless, regarding all these attempts, it is throughout the paper that I'm pursuing the idea that most of them, including Edgington's theory, either fail¹ in their intention to vindicate verificationism, or, intending to block the effect of Fitch's argument in the framework of non-classical (intuitionistic, relevant, but most often para-consistent and para-complete) logics, do so by introducing principles that invalidate some of the basic classical rules and principles. In any case, there is no decisive evidence that any of the non-classical approaches can help verificationists escape the grip of FA.

¹It is interesting that some of the leading logicians suggest different non-classical logics as the only way for blocking FA. For instance, Williamson claims, "How else might a verificationist escape from Fitch's argument? One way would be to substitute intuitionistic for classical logic. It may even be the only way" (Williamson, 1987, 261). Priest claims something quite different, "We have seen that Fitch's argument may be blocked by an appeal to dialetheism. Moreover, it is the *only way* [my italic] that we have found in which the argument may be blocked (In Salerno, 2009, 100–101). However, I'm suggesting that there is no plausible way to vindicate verificationism.

However, before entering the topic more deeply, a few notes concerning the general importance of the argument are required. What is nowadays known as Fitch's argument² (FA) is also referred to as the knowability paradox by many authors. In this form, it is an unavoidable topic in contemporary formal epistemology, particularly in the context of what, in principle, one can know. The key question in this regard is: *are all true propositions knowable?* Answering this question, realists and anti-realists in epistemology determine their opposing positions. Realists, holding that humans are non-omniscient, answer negatively. On the other hand, anti-realists answer positively: if something is true then it can be known. Let us call this the *knowability thesis* (KT). Formally:

$$(KT) \forall \varphi (\varphi \supset \diamond K\varphi).$$

A much stronger, but unreasonable, even *silly*³ form of verificationism is that all truths are in fact known:

$$(SV) \forall \varphi (\varphi \supset K\varphi).$$

Both variants of verificationism should endorse omniscience concerning the knower. The alternative is, quite generally, that there are no omniscient agents and that for any agent S there is the truth she does not know (not all truths are known) and that possibly no one will ever know. Let us call this claim *non-omniscient*. Formally:

$$(\text{Non-omniscient}) \exists \varphi (\varphi \wedge \sim K\varphi).$$

The (KT), $\forall \varphi (\varphi \supset \diamond K\varphi)$, is at the heart of verificationism. Furthermore, it is often considered to be the *quintessential implication* of semantic anti-realism⁴ (Kvanvig, 2006, 56).

² What I call Fitch's argument is usually referred to in the literature as Fitch's or Church-Fitch's paradox. Some authors, however, deny that the argument is paradoxical (see Williamson, 2000), claiming that it is only surprising. To avoid this discussion, I prefer a more neutral term, Fitch's argument.

³ Williamson (2000, 272) illustrates the "silliness" by inviting us to imagine a situation in which his office contained either an even number of books at some time t in the past or not. Nobody knows, as a matter of contingent truth. Thus, either it is an unknown truth that it was an even number of books at t , or it is an unknown truth that it was an odd number. Either way, there is an unknown truth and strong verificationism is false.

⁴ The knowability thesis follows immediately from the verificationist's claim that a proposition φ is true if and only if it is possible to prove (or verify) φ . If it is possible to prove φ , it is possible to know that φ .

As mentioned, verificationists should accept the positive answer to the question, “are all truths knowable?”. In short, verificationism constrains truth epistemically. It equates truth (or meaningfulness) with a cognizer’s cognitive ability, an ability to know, to believe, to verify, to confirm. Realism is, on the other hand, ontologically committed. Some object (a real number, for example) exists independently of our cognitive ability to grasp them. This independence of reality from our cognitive grasping limits our knowledge in the sense that there are unknown, even unknowable propositions. We are non-omniscient both in the mathematical realm (Gödel’s undecidability) as well as in the realm of contingent, non-mathematical propositions. Accordingly, realism is bound to hold that some truths are not known and some of them possibly not knowable.

There are various reasons for accepting anti-realism. Besides well-founded motivations in the history of philosophy, issuing either from the intention to avoid scepticism (from Berkeley to American pragmatists) or from the “meaning as use” doctrine (Wittgenstein, more recently Dummett), the optimistic idea that all truths are knowable, at least in principle, is one of the tenets deeply entrenched in the modern scientific worldview, which is rarely questioned. It is widely accepted by physicalists (to be more precise, the knowability thesis fits particularly well with materialists or methodological physicalists, as Kvanvig named them (2006, 43–47)). The modern idea of verificationism goes back to American pragmatists and logical empiricists. C. S. Pierce, for instance, claimed that truth is what the scientific community would agree on in the long run. For logical empiricists, truth (or meaning) is tied to what we are capable of verifying. In the long run, the proponents of the modern worldview firmly believe that all truths will be known and that the important problems will be ultimately solved.

The optimistic standpoint is, of course, worth holding, but its epistemic background is doubtful, at least according to FA. The moderate verificationism (expressed as KT) gets in trouble precisely because FA presents a relatively simple proof, given in a few lines, arguing that verificationism is inconsistent. This is certainly bad news for the anti-realists’ optimism. Namely, the claims that all truths are knowable is provably equivalent to the omniscient-like claim that *all truths are known*. Let us call it the knowability principle (KP). We can put it formally:

$$(KP) (\varphi \supset \Diamond K\varphi) \vdash (\varphi \supset K\varphi).$$

The claim on the left of the formula (KT), or, as Kvanvig notes, “the quintessential implication of the semantic anti-realism”, is equivalent to the problematic claim on the right side of the formula that all truths are, in fact, known (Williamson (2000) calls this claim strong verificationism). This claim seems unreasonable, even silly.

This devastating effect of FA, of course, puts convinced anti-realists (verificationists) on high alert, but also unsettles those who feel inclined to the appeal of realism, however, at the same time hold that the optimistic worldview is worthy enough to be vindicated⁵. Therefore, it is not surprising that van Bentham (2004, 105) says, “Much of the literature on Fitch’s Paradox seems concerned with averting a disaster, and saving as large a chunk of verificationism”. Faced with this peril, a verificationist has a choice. She might try to save as much of verificationism as possible by formulating the counter-argument either in the framework of classical⁶ (CL) or in the frame of the non-classical logical (NCL) systems. A suitable tool seems to be intuitionistic logic. Namely, as Williamson claims (1987), KP can hardly be refuted by means of classical logic, but with the extended operators in intuitionistic logic, it might be possible. However, in this paper, I claim that despite numerous endeavours, observing them generally, various projects of saving verificationism expressed in the slogan “what is true can be knowable”, do not succeed. It is arguably so concerning vindications as expressed in the frame of classical logic, whereas when expressed in non-classical (para-complete and/or para-consistent) logics, they can block the effect of FA, but at the price of postulating trivial worlds. This is certainly unacceptable for ones who endorse classical logic. However, it might also be too costly for a verificationist who accepts a non-classical logic.

In what follows, FA is going to be briefly presented. This paragraph is followed by an overview of different strategies that aim to block, in a way, the threat of FA for anti-realism. There are numerous attempts to refute or at least minimize the effect of FA on verificationism. I’m proposing to divide them into two main groups of strategies, the *restrictionist* and the *revisionist* ones. The restrictionist strategy, exemplified by its most important representative, Dorothy Eddington, is presented at length and in more detail, while the revisionist ones are briefly indicated.

⁵ This uncomfortable situation is nicely put forward by Šuster: “Although realism is close to me, I nevertheless think that moderate anti-realism, which represents a complex set of concepts and belongs to the historical treasury of philosophical ideas, goes with a developing conceptual complex, and it is hard to believe that it will be buried by a few lines of normal modal logic” (Šuster, 2023, 92, translated from Slovenian by the author).

⁶ By classical logic I mean the propositional and predicate calculus, but also normal modal and epistemic logic.

2 Fitch's Argument

Let us start with the informal characterization of the argument. Let us take that every true proposition is knowable, and suppose, for reductio, that there is a proposition, φ , which is true but not known, $\varphi \wedge \sim K\varphi$. Then it must be possible to know $\Diamond K(\varphi \wedge \sim K\varphi)$. It is now easy to prove that it is possible to both know φ and not know it, $\Diamond(K\varphi \wedge \sim K\varphi)$, which is a contradiction (see [13]).

Before presenting the formal proof, let us recall the two variants of verificationism mentioned above, both supposing the *knowing agent to be omniscient*. One of them is the knowability thesis, a reasonable assumption, that all true propositions can be known by someone sometime:

1. $\forall \varphi (\varphi \supset \Diamond K\varphi)$, *knowability thesis (KT)*.

The other, less reasonable, if reasonable at all, is that all truths are (actually) known. This strong form is obviously false and unacceptable, similar to Berkeley's solipsism (*esse est percipi*):

2. $\forall \varphi (\varphi \supset K\varphi)$, (*SV*) or "*silly*".

In opposition to these assumptions is the realistic, non-omniscient idea:

3. $\exists \varphi (\varphi \wedge \sim K\varphi)$.

The various forms of proof proceed by employing additional assumptions. Omitting the quantifiers, we can group them as follows and formalize them accordingly:

Epistemic rules

(Fact): $K\varphi \supset \varphi$, saying that *knowledge is necessarily factive*.

(Dist): $K(\varphi \wedge \psi) \supset (K\varphi \wedge K\psi)$, *distribution over conjunction*.

⁷ Wherever the argument is presented in the setting of classical logic, I am using the "horseshoe", indicating material implication. Always when the argument is treated non-classically, the arrow sign (\rightarrow) is used.

Alethic modal rules

(LNC): $\sim\Diamond(\varphi \wedge \sim\varphi)$ ⁸, a *law of noncontradiction*.

(Close): $(\Diamond\varphi \wedge (\varphi \supset \psi)) \supset \Diamond\psi$, modal formulation of *closure principle*.

The rule of inference is

$\vdash\varphi \supset \Box\varphi$, *Rule of Necessitation*.

The proof,⁹ briefly presented, proceeds as follows. It starts with two contrasting propositions, *KT* ($\varphi \supset \Diamond K\varphi$) and (unknown) $\varphi \wedge \sim K\varphi$. Substituting the (unknown) in (KT) as the value of φ , we get, by modus ponens:

1) $\Diamond K(\varphi \wedge \sim K\varphi)$.

Assuming $K(\varphi \wedge \sim K\varphi)$ and applying (*dist*) over conjunction we get:

2) $K\varphi \wedge K\sim K\varphi$.

By application of the *Fact* to the second conjunct of 2), it yields:

3) $K\varphi \wedge \sim K\varphi$. By *reductio*:

4) $\sim K(\varphi \wedge \sim K\varphi)$.

Application of the Rule of Necessitation, $\vdash\varphi / \Box\varphi$, yields:

5) $\Box\sim K(\varphi \wedge \sim K\varphi)$.

⁸ Given that the dual of $\Box\sim\varphi$ is $\sim\Diamond\varphi$, we have: $\vdash\sim\varphi \vdash \sim\Diamond\varphi$.

⁹ In presenting FA, I am following Kvanvig's simple and elegant formalization. See slightly different formalization in Wansig's (2002):

1) $\varphi \wedge \sim K\varphi$	assumption
2) $\Diamond K(\varphi \wedge \sim K\varphi)$	1, WV
3) $\Diamond(K\varphi \wedge K\sim K\varphi)$	2, Dist
4) $\Diamond(K\varphi \wedge \sim K\varphi)$	3, A3
5) $\sim\Diamond(K\varphi \wedge \sim K\varphi)$	A3
6) $\sim(\varphi \wedge \sim K\varphi)$	1, 4, 5

Expressing $\Box \sim \varphi$ by its dual, $\sim \Diamond \varphi$, we eventually have:

$$6) \sim \Diamond K (\varphi \wedge \sim K\varphi)$$

Line 6 contradicts line 1. The verificationists must deny that there are truths we do not know (that we are non-omniscient), which leaves us with:

$$7) \sim \exists \varphi (\varphi \wedge \sim K\varphi). \text{ The conclusion is that all truths are actually known:}$$

$$8) \forall \varphi (\varphi \supset K\varphi).$$

Showing that KT collapses to SV, FA brings verificationism into trouble. But does this result conclusively refute any possible vindication of verificationism? Is it fatal for verificationism? Many think it is not. As Kvanvig says, there is a long way from Fitch's argument to the refutation of any form of verificationism, and, expectedly, many philosophers have sought to free verificationism from the commitment to KP $(\varphi \rightarrow \Diamond K\varphi) \vdash (\varphi \rightarrow K\varphi)$ in order to avoid a refutation of anti-realism by FA.

3 Can verificationism be vindicated?

To provide an overview of the recent discussion concerning possible avoidances of the effect of AF, as well as their critics, on verificationism, let me utilize C. Jenkins (in Priest, 2009, 304) who offers a succinct formulation of the given problem in three questions:

- i) Does Church–Fitch's argument really refute global anti-realism?
- ii) If it does not, is this because the argument is fallacious, or because anti-realists are not in fact committed to KT¹⁰?
- iii) If anti-realists are not committed to KT, how should their doctrine of epistemic accessibility¹¹ be expressed?

¹⁰ KT $(\varphi \supset \Diamond K\varphi)$ Jenkins calls WVER, weak verificationism, following Williamson.

¹¹ By the doctrine of *epistemic accessibility*, Jenkins (in Salerno, 2009, 302) means the anti-realist idea that "because of its mind-dependent nature, all of reality is epistemically accessible to us".

To answer the question negatively, *i*) would obviously be too hasty, so we are going to assume a positive answer, namely that FA really refutes verificationism. In this case, the presentation of the problem regarding *ii*) might proceed either to claim a) that the proof procedure in FA is erroneous or to come back to b) claiming that verificationism is not, in fact, committed to KT ($\varphi \supset \Diamond K\varphi$). Concerning *iii*), we agree with Kvanvig, who claims that “the logic of the paradox is not in any simple way problematic” (2006, 14). However, to inspect more closely where the proof actually might go wrong, one can locate it either in the very steps of the proof or one can cast doubt on the additional assumptions (*epistemic, alethic-modal rules and the rule of necessitation*). The steps of the proof are doubtlessly correct. Concerning the *assumptions*, it turns out that it is (dist) *the distribution over conjunction* that some authors indicate as a weak point. Williamson (2000), for instance, has claimed that knowledge need not be distributed over conjunction, but as a full-blooded realist he convincingly claims that this can hardly help the verificationist in avoiding FA because “the anti-verificationist argument can be reconstructed in at least two ways¹². The verificationist cannot escape by denying distribution” (2000, 84–85).

This leaves us with the second disjunct of *ii*), that verificationist (anti-realist) is not, in fact, committed to KT. This being the case, the answer to question *iii*) requires a kind of taxonomy of various strategies that hope to resist the threat of FA. A rough taxonomy that more or less corresponds to what most of the authors (see Brogaard & Salerno, 2019, 2002; Jankins, 2009; Kvanvig, 2006) propose is to divide various attempts into two groups. The widely accepted division is on *restriction* and *revision* strategies. According to my understanding of the taxonomy, *restriction* strategies mostly accept classical logic (CL) as a framework for dealing with FA, while the very strategy consists of restricting the scope of the universal quantifier in: *for all x, x can be known*. A bit more specifically, (Jankins, 2009, 305), “not all true propositions are supposed by anti-realist to be knowable, but only some”. In this paper, considerable attention will be paid to Edgington’s proposal, which suggests that only *actual* truths are knowable. I’m going to show, relying on the criticism of the thesis (Williamson, 1987; Percival, 1990) the proposal does not succeed.

¹² Relying on two variants of verificationism, the weak ($p \rightarrow \Diamond Kp$) and the strong one ($p \rightarrow Kp$), he (Williamson, 1993, 84) claims that one can reconstruct the anti-verificationist argument without relying on distribution, either by a) arguing that verificationists are committed to something stronger than ($p \rightarrow \Diamond Kp$) or by b) deducing something weaker than ($p \rightarrow Kp$) from ($p \rightarrow \Diamond Kp$).

Revisionist strategies, on the other hand, concern the question of whether the “proper” logic of knowability is the classical logic and, if not, whether the substitution of classical logic with some non-classical logic (NCL) can, by invalidating Fitch’s reasoning, help the verificationist to vindicate her standpoint? Given that FA stands or falls with the logical principles we referred to as the *additional assumptions* (epistemic, alethic-modal rules and the rule of necessitation) that are rendered as valid in the framework of CL, while some of them (or all) are invalidated in different forms of NCL, the revisionist typically considers CL as inappropriate for avoiding FA. The NLC logics proposals are of the paracomplete or the paraconsistent kind. Concerning the intuitionistic logic, in which several important verificationist attempts (Dummett, Tennent) are grounded, it should be noted that it is counted as paracomplete, holding that p and $\sim p$ can both be false. It invalidates the law of excluded middle (LEM: $\varphi \vee \sim\varphi$). The question is whether substituting CL with NCL can help verificationism in vindicating its standpoint: *all truths are knowable*.

4 D. Edgington: Trans-worlds knowability

In the restrictionist camp, Dorothy Edgington (1985) ingeniously proposed a solution to the devastating effect of FA, offering a modification of the verification principle, $(\varphi \supset \diamond K \varphi)$. Accepting that in the present setting, KP is inconsistent with $(\varphi \wedge \sim K \varphi)$, she introduced a variant reading of the claim “every truth is knowable”, arguing that, under a *suitable interpretation*, the assumption that all actual truths are knowable, $(\forall \varphi) (\varphi \supset \diamond K \varphi)$, and the assumption that some actual truth is not known, $(\exists \varphi) (\varphi \wedge \sim K \varphi)$, are consistent. The crux in Edgington’s endeavour is to make the distinction between the situation in which *one knows* and the situation *one knows about*. In light of this twist, as Williamson observes, the trouble with KP “is that it conflates the situation s in which p with the situation s' in which it is known that, *in s*, p ” (Williamson, 1987, 256). To offer a brief exposition of Edgington’s understanding of *suitable interpretation*, a few preliminary steps are needed. It should first be noted that, instead of possible worlds, she speaks of the concept of “possible situations,” which are close enough but less specific than the “possible worlds” concept. Next, to discriminate the situation in which *one knows* from the situation in which *one knows about*, she introduces a new, tense operator S. Applied to the contradiction $(\varphi \wedge \sim K \varphi)$, the new form, SK $(\varphi \wedge \sim K \varphi)$, is introduced, where S means “it will be or is or was the case that.” The main idea can now be presented.

Expressed in the notation of *possible situations* (let's call this *quantifying context*, QC), the above claim can be formulated as:

$$(QC) \forall s ((\text{in } s, p) \rightarrow \exists s' (\text{in } s', K(\text{in } s, p))),$$

meaning that, for every situation s , if p is true in s , it is known in s' that p is true in s , (Edgington, 1985, 367). Now, there is no reason “why it should not be known in s' that in s , it is unknown truth that p ” (as Williamson formulates this in 1987, 256).

However, the situation is not as simple as that. There is a problem arising in this proposal and it concerns the relationship between knowledge in two kinds of situations, in situation s' , “from” which one observes the truth, and situation s where the observed truth is located. Namely, the knowledge in situation s' and in situation s must be the same knowledge, which apparently is not the case.¹³ Attempting to solve the problem, Edgington seems to see a way out by introducing a temporal analogy to QC, where instead of evaluating sentences at situations (s and s'), tense sentences should be evaluated at time points t and t' . Let us call this the *temporal context*:

(TC) For every p , if p is true at t , then $(\exists t')$ (someone knows at t' that p is true at t).

This formulation, hopefully, makes it easier to establish the relationship between knowledge contexts in the temporal analogy than is the case in the quantified, QC variant. This being the case, the claim goes like this: if the relationship between, for instance, the thought “It was raining” expressed at seven o'clock and the thought “It is raining” expressed at six o'clock can be established (in TC), then it can be established for the relationship between the situations s' and s , in QC formulation.

To accomplish this brief review of Edgington's proposal, the above quantification over situations should be reconciled with the original *modal version* of KP, $(\forall \varphi (\varphi \supset \Diamond K\varphi))$, the principle she wants to vindicate. To do that, Edgington equates the

¹³ The only characteristic of the concept of knowledge we need at this point is its standard realistic account, namely that knowledge is factual. That means that an agent s knows that p only if p . Accordingly, two persons have the same factual knowledge iff they know the same factual contents, and their content-bearers (thought, expressed sentence) express the same factual content iff the content-bearers have the same truth-conditions. This is the case iff they would be made true by the same fact where they are both true. It is obvious that in our situations, s and s' knowledge can not be the same because the facts known are different (compare Percival, 1991)

situation s (in QC, or time point t , in TC), the situation in which one knows that p , with the *actual* situation designated with the operator A . In terms of possible worlds, it is now consistent with the claim that someone in some other world (call it *non-actual* world w) knows that in the *actual* world, w_A one knows a proposition. In formal notation, we get: $\Diamond KA(\varphi \wedge \sim K\varphi)$, meaning that there is some world w in which it is known that it is true in the actual world w_A that φ is true but not known (compare Kvanvig, 2006, 57). Finally, it is suggested that the contradiction can be resolved by:

(A) $A\varphi \supset \Diamond KA\varphi$. This seems to be consistent. But is it so?

However, there are several convincing criticisms concerning Edgington's argument, we will pursue two of them. Williamson's (1987) and Percival's (1991) criticisms seem to be particularly compelling. Williamson's criticism, in fact, identifies three weak points in Edgington's proposal. The first one has to do with the difference between necessary and contingent propositions. Both are supposed to be known (by the agent) in most versions of verificationism, but it is dubious whether it is so in Edgington's proposal. The second issue deals with the problem of identification of knowledge across worlds (or situations). For the formulation in (A) to work, the knower should have the same content of knowledge at the actual and at the non-actual situation (world). However, this seems to be problematic as well. The final point aims to challenge the supposed analogy between the temporal knowledge in TC (knowledge identity across time) and the knowledge in modal contexts (knowledge identity across worlds) in which the proposal is formulated. The two last points are common targets to both, Williamson's and Percival's criticism.

a) The crucial question for the first critical point refers to the kind of propositions that are supposed to be knowable according to the same (A), $A\varphi \supset \Diamond KA\varphi$. According to the original, *knowability principle* ($\varphi \supset \Diamond K\varphi$), the obvious answer is that such propositions are to be *contingent* as well as *necessary* ones. However, Edgington's defence of verificationism by transforming ($\varphi \supset \Diamond K\varphi$) to ($A\varphi \supset \Diamond KA\varphi$) represents, as Williamson (1987, 257) claims, a "surprisingly weak form of verificationism". Namely, $A\varphi$ is true at s (or at w in the modal variant) iff φ is true in the actual situation (world). To be true in the actual, φ has to be true in all accessible situations (worlds). Therefore, $A\varphi$ and $\Box A\varphi$ are equivalent. Thus, Williamson (1987, 258) claims, "In s, p' , (for most values for s and p) is therefore

necessarily true, if true at all”. Therefore, it follows that this variant of verificationism is committed only to the *knowability of necessary truths*. This being the case, verificationism would not be in a position to require any *contingent* truth to be knowable¹⁴.

b) It has been mentioned that the crucial idea of Edgington’s proposal is that all truths can be known only if a truth is known ”from” a situation other than that in which it is located. The schema (A) claims that it is a *non-actual* situation (world) from where a proposition *p* in an *actual* situation is known. The difference between situations is crucial because without determining the relationship between actual and non-actual knowledge, the scheme (A) would be in danger of collapsing “into the obviously silly schema $A\phi \rightarrow AKA\phi$ ” (Williamson, 1987, 260), in which case Edgington’s solution would be back to the initial problem. Having established the difference between situations, a clear account for the *relationship* between thoughts (taking it to be a content-bearer of knowledge) in the non-actual and the actual situation should be characterized. Even more, the defence of the schema (A), $A\phi \rightarrow \diamond KA\phi$, depends on giving such an account. But that is exactly where the problem lies.

Let the truth *p* in our example be “the earthquake of the low intensity happens in a situation *s* and no one knows that”. In terms of QC, it can be known “from” the non-actual situation *s*’. The formulation QC requires that the truth that *p* expresses in *s* is the same as the truth which “In *s*, *p*” expresses in *s*”. The problem is to

¹⁴I am grateful to an anonymous referee who drew my attention to the Rabinowicz’s and Segerberg’s (1994) paper proposing a response to the criticisms of Edgington’s version of verificationism. Due to the space limitations, I will only briefly present their views. Particularly, my focus is on whether their proposed response threatens Williamson’s first objection to Edgington’s proposal. As is claimed, Williamson (1987) argues that Edgington’s variant of verificationism is committed to the knowability of necessary truths only. Namely, according to the standard truth-conditions for actuality, $A\phi$ is true at *w* iff it is true at all actual worlds. Therefore, actuality is equivalent to necessity, $A\phi \leftrightarrow \Box A\phi$. Rabinowicz and Segerberg recognize the seriousness of the problem and admit that it cannot be solved with standard semantics’ resources. The source of the problem, they claim, lies in the inability of standard semantics (standard truth-conditions), used by Edgington “to mix the actuality operator and the epistemic operator” (1994, 104). In standard semantics, the perspective from which one knows that *p* (actual world) is true in the non-actual world has been considered as fixed. Rabinowicz and Segerberg have proposed a solution that consists in the introduction of utterly new semantics, such that the standard semantics with the fixed actual world is replaced with the two-dimensional, variable-perspective semantics (1994, 104). In the two-dimensional semantics, “a formula is being evaluated not just at one point, *v*, but at an ordered pair of points, (*w*, *v*), with *w* being the point of perspective and *v* the point of reference.” [1994, 104]. However, the implementation of new semantics, variable-perspective ones, changes in a considerable manner the meaning of the necessity operator, as well as the actuality and knowability operator. As a consequence, it is only under *this interpretation* that the solution to FA (the denial of the knowability principle) seems to be plausible. Accordingly, Rabinowicz’s and Segerberg’s analysis cannot be considered as a treat to Williamson’s criticism because it is entirely situated in standard semantics.

determine what counts as the same knowledge in respectively different situations (or worlds). Percival (1990) calls this problem the “knowledge-identity across contexts”. Edgington, being, of course, aware of this, admits that if, for instance, an agent A had non-actually had a thought, expressed in words “it is actually the case that p ”, A would not have been expressing the thought of the requisite kind, since his use of “actually” would refer rigidly to his own situation, not to B’s. Without going much deeper into it, Edgington hopes that it can be resolved if a) the analogy between the modal (expressed either in terms of possible situations or in worlds) and temporal formulation (TC) can be established and b) if the account of knowledge between non-actual and actual situation can be given in the temporal context, it is also justified in the modal context. The criticism in both Williamson (1987) and Percival (1991) admit that the analogy between the modal and the temporal can be established. But even if that is the case, the suitable relationship between knowledge in timepoints t and t' can be claimed only if there is a causal relation between knowledge in t and t' . However, the same relationship does not hold in the modal context, and therefore, the argument that (A) can refute KP fails.

The brief reconstruction of Edgington’s requirement a) can go like this: the requisite relationship between non-actual and actual can be re-established in the frame of the temporal context (TC), in terms of “now” (the temporal analogy of the non-actual situation; for the sake of example, at seven o’clock) and “then” (actual situation; at six o’clock). Although agent A, at seven o’clock, would not be able to express the thought “it is now raining (six o’clock)”, since A’s use of the word “now” will refer to seven o’clock and not to six o’clock, the relationship between the thought at seven o’clock about the thought at six o’clock can be saved if two thoughts were casually connected. Williamson (1987) admits that there can be such a connection and specifies the possible connection in this way: “The only apparent way in which one might think at seven o'clock the thought that one could have expressed at six o'clock in the words 'It is now raining' is by remembering six o'clock” (1987, 257). But, he continues, “Though it might be that the causal link of remembering can work in temporal context, there is no causal link between the non-actual and the actual” (1987, 258). Williamson’s point is that “there is an insuperable fact that this analogy does not work in the case of ”actual” and ”non-actual” (1987, 258). A similar point is made by Kvanvig (2006, 57–58):

“[I]t is questionable whether it is possible to entertain any proposition about some singular, individual world not identical to the world one is in. Any way of describing the world will apply equally to a number of different worlds, and any sort of different reference to other worlds would seem to be impossible in virtue of lack of any causal link between possible worlds.”

In addition to this, Percival (1991) gives a sophisticated and long exposition of why it does not work. We will quote at this point only his indication of why Edgington’s endeavour does not succeed:

“I think she is misled by the formal similarities between standard semantics for tense- and modal-logics. But in reasoning analogically from the temporal to the modal case she does emphasize that the knowability principle and its temporal analogue, the thesis that all present truths were, are, or will be known, cannot be assessed without establishing what counts as the same knowledge in, respectively, different worlds and times.” (Percival, 1991, 82–3)¹⁵

It is reasonable to conclude this section of the paper by emphasizing that Edgington’s, I would say a heroic attempt to vindicate verificationism against FA in the framework of classical logic, did not succeed.

Revisionist strategies

The family of *revisionist* strategies concerns primarily the problem of the proper logic for the knowability paradox. According to my classification, what is meant by *revisionist strategies* are those attempts aiming to undermine FA, which replace classical logic (CL) with some of the non-classical logics (NCL). The line of demarcation between revisionist and restrictionist strategies concerns, therefore, a kind of logic supposed to be suitable for weakening the FA effect on verificationism. To avoid any misunderstanding, it should be noted that a particular revisionist strategy may

¹⁵ As a response to criticisms (particularly concerning Williamson’s problem b)), Edgington published a paper (2010) in which she strengthened the arguments made in the original paper (1985) and offered new ones. She further develops her argument for counterfactual knowledge (in 1985, 563). Williamson’s answer to Edgington’s response followed in 2021. Concerning the argument from counterfactual knowledge, which, according to Edgington, blocks FA, Williamson argues that the argument is subject to trivialization. To avoid trivialization, Williamson claims, Edgington needs to offer a more general account of how the knower is allowed to specify a counterfactual situation for the purposes of her argument, and it is unclear how to do so.

be restrictionist in character in the sense that it proposes the restriction of the scope of the knowledge operator, but making it in terms of NCL. The restrictionist strategy, on the contrary, restricts the scope of the knowledge operator, making it in the terms of CL.

The NCL that we are interested in are either paracomplete or paraconsistent,¹⁶ or both. Among those non-standard logics, the *intuitionistic modal logic* (IML) is particularly suitable for most kinds of anti-realism,¹⁷ exceptionally so in Dummett's¹⁸ (1977) interpretation. As Percival (1990) notes, "Dummett has repeatedly argued, on grounds independent of Fitch's proof, that anti-realism warrants intuitionistic rather than classical logic and an assertability-conditional rather than a truth-conditional theory of meaning." (1990, 182–183). One of the prominent differences between classical modal logic and IML is the reinterpretation of the operator K (which in CML means "it is known that") as "it is verified that". Adding the possibility operator \diamond , the idea is expressible (in Williamson, 1992) as:

φ iff it is possible that it is verified that φ .

This general intuitionistic formulation, applied to the context of FA, substitutes the knowability thesis, $\varphi \supset \diamond K\varphi$, becoming $\vdash \varphi \leftrightarrow \diamond K\varphi$ (expressing exactly: φ iff it is possible that it is verified that φ). In this interpretation, however, the possibility operator \diamond changes and takes on a peculiar meaning. Namely, the schema $\vdash \varphi \leftrightarrow \diamond K\varphi$ yields a "disastrous schema $\vdash \diamond \varphi \rightarrow \varphi$ " (see Williamson, 1992, 67), where the notion of possibility collapses into actuality.¹⁹ We are not going to go deeper into the characteristics of IML, but will instead turn to one prominent representative of the intuitionistic interpretation of KT. We are going to briefly present the main points in Neil Tennant's proposed solution for blocking the knowability principle $((\varphi \supset \diamond K\varphi) \vdash (\varphi \supset K\varphi))$.

¹⁶ Loosely speaking, a paracomplete logic is a logic in which a proposition and its negation can both be false, while paraconsistent is the one in which a proposition and its negation can both be true.

¹⁷ It, for example, nicely accommodates Putnam's *internal realism*.

¹⁸ In his works, Dummett (2001) does not explicitly address Fitch's paradox in an intuitionistic setting, but he offers a solution to KP without relying on intuitionism.

¹⁹ Schema $\vdash \diamond \varphi \rightarrow \varphi$, which is highly controversial, is the exact opposite of the non-controversial schema $\vdash \varphi \rightarrow \diamond \varphi$, meaning: if φ obtains, it is possible.

Here is a brief indication of his (Tennant, 1997) variant of an intuitionistic answer to FA. It consists of the modification of the knowability thesis (KT), in which the epistemic operator K is restricted. The main idea of the restriction strategy is that not all but only a restricted domain of propositions is knowable. The proposition to be knowable should be *Cartesian*, and a proposition φ is such if and only if the contradiction does not follow from $K\varphi$. This claim is summarised in Brogaard & Salerno (2002, 146) as:

Every true statement A is knowable, where ‘ $K(\varphi)$ ’ is not self-contradictory.

Williamson (2009) formalises this statement as follows:

$(\Diamond KC) \varphi$; ergo $\Diamond K\varphi$, where φ is Cartesian.

Informally, $(\Diamond KC)$ says that truth entails knowability except when Fitch’s problem occurs. This proposal seems to be all too simple. As it is said by Brogaard & Salerno (2002), “A defence of this clause is all that is needed to block the problematic substitution of ‘ $B \ \& \ \sim K(B)$ ’ for ‘ A ’ in the knowability principle. After all, $K(B \ \& \ \sim K(B))$ is self-contradictory”. Tennant’s restriction was subject to numerous criticisms, for instance, by Brogaard & Salerno (2002), Kvanvig (2006), and Williamson (2009). At first glance, many would agree that the idea of a Cartesian restriction of the epistemic operator is “desperately”²⁰ ad hoc. Whether or not this is the case, critics raise other, more severe objections. Since it is beyond the purposes of this paper to give an overall assessment of Tennant’s proposal, I’m just going to indicate a general direction of the criticisms.

It is apparent that, for $(\Diamond KC)$ to hold, it should be decidable for every φ whether it is Cartesian or not. In brief, the claim, “there is no undecidable statements”, must hold. The undecidable statement (neither it nor its negation is known) is of the form: $\sim K\varphi \ \& \ \sim K\sim\varphi$. However, the problem of decidability, as well as the intuitionistic meaning of the possibility operator \Diamond , are the focus of critics (see Brogaard & Salerno, 2002; Williamson, 2009).

²⁰ See Williamson, 1993.

Pointing out the general idea of the criticisms, let me remind you of the “disastrous” schema $\vdash \Diamond \varphi \rightarrow \varphi$, which is a standard anti-realist meaning for \Diamond , making possibility factive. In epistemic reading, it is: $\Diamond K\varphi \rightarrow \varphi$. By contraposition, we get: $\sim \varphi \rightarrow \sim \Diamond \varphi$. It is easy from this to get: $K\sim \varphi \rightarrow \sim \Diamond \varphi$ (comper proof in Brogaard & Salerno, 2002). Hence, this form of anti-realism entails that there are no undecidable statements. However, $(\Diamond KC)$ requires that it should be decidable for every φ whether it is Cartesian or not. Since one cannot know in advance whether φ is Cartesian, anti-realism is faced with a kind of paradox of decidability. Concluding their long analysis of Tennant’s proposal, Brogaard & Salerno (2002) state that “the restriction strategies proposed thus far are insufficient to treat the real problem”.

To conclude the discussion concerning strategies for avoiding the effect of FA on verificationism, the remaining part is a paraconsistent approach. Those logics, certainly important and intriguing as they are, still cannot significantly contribute to the vindication of verificationism. Due to the space limitation, we are going only to gesture toward its proposed solutions. Generally speaking, paraconsistent approaches suggest appealing either to the *truth-value gap* or to the *truth-value glut* to block the effect of FA on verificationism. In either way, the verificationist principle can be proved as valid. However, invalidating some of the relevant rules in Fitch’s proof (in Priest’s *dialetheist* proposal, for instance, it is the rule of *contraposition* (see Priest, 2009), and in proposed semantics introducing *trivial* worlds, $\alpha \vdash \Diamond K\alpha$ becomes *vacuously* valid and verificationism is (vacuously) vindicated. In principle, the same holds for other paraconsistent approaches (for example in Biell, 2009). It should be noted that appealing to the truth-value gap or the truth-value glut (or both, in some combinations) can block *any* proof or argument. The question is whether this manoeuvre of making the verificationist principle vacuously valid can really help the verificationist to win the battle against the FA challenge.

In any case, the proof supporting the knowability principle, $(\varphi \supset \Diamond K\varphi) \vdash (\varphi \supset K\varphi)$, valid in classical logic, is robust enough to resist any attempts, formulated in the frame of classical logic, to refute it. Approaches coming from the camp of non-classical logics can invalidate the knowability principle, but it is dubious whether they, in fact, can help anti-realism. Let me conclude with a much sharper verdict. As Williamson (1993, 204) notes, “The attempts on behalf of anti-realism to deal with the Fitch problem give every sign of a degenerating research programme.”

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