Theoretical calculation of stress for the start of stress induced martensitic phase transformation in the Shape Memory Alloys NiTi

Teoretični izračun napetosti za začetek napetostne martenzične fazne transformacije v zlitinah s spominom oblike NiTi

Janko Ferčec¹, Rebeka Rudolf¹,²

¹ University of Maribor, Faculty of Mechanical Engineering, Institute of Materials Technology, 17 Smetanova Str., 2000 Maribor, Slovenia
² Zlatarna Celje d.d., 19 Kersnikova Str., 3000 Celje, Slovenia
E-mails: janko.fercec@uni-mb.si; rebeka.rudolf@uni-mb.si

Abstract: The Shape Memory Alloy Nickel – Titanium NiTi is characterized by particular thermo-mechanical behaviour. Namely, the NiTi alloy showed above the temperature where the fully austenitic phase behaviour has superelasticity. Superelastic behaviour is a property of the material and means that the material during the loading at a given stress causes the diffusion-less phase transformation from austenite to martensite. In doing so, a small rise in stress in the material causes changed elongation. The value for the beginning of transformation in the Shape Memory Alloy NiTi is highly dependent on the method of loading. Under tensile and compressive loading, asymmetrical stress occurs. In the present work we showed experimentally determined stress for the start of the transformation at tension loading and the theoretical calculation of the stress to start the transformation for compressive and shear loading.

Key words: NiTi alloys, shape memory, superelasticity, stress.

1. Introduction

The Shape Memory Alloy (SMA) Nickel titanium (NiTi) in recent decades has become a much used material for commercial purposes, especially for medical applications. One of the first uses of SMA NiTi was in orthodontic praxis for fixed orthodontic appliances. The part to be used from this material is a wire the function of which is to operate on the teeth with a force. A very important factor for the use of various materials in medicine is that they are not toxic to the body. Due to the ability to form a titanium oxide layer on the surface of the wire this SMA NiTi has good biocompatible properties. Another very important factor which makes the SMA NiTi very useful in orthodontic practice is its unique functional properties [1].

The property or behaviour that makes this SMA NiTi very useful in orthodontic treatment is called...
superelasticity. Superelasticity means the ability of SMA material to return from a very large strain back to its original shape. This is associated with diffusion-less phase transformation in the solid. At a loading of SMA (tensile loading Figure 1) in the material phase transformation begins from the initial or austenitic phase into a detwinned martensitic phase. During this change (points B to C) a major change in strain occurred and a small change of stress. Compared to steel NiTi SMA has a much larger recoverable strain, which makes this material very useful in the initial stage of orthodontic treatment. Namely, where there are large deflections of a tooth a large deformation of this wire is necessary [2 - 4]. Another functional advantage of NiTi SMA is in the fact that this material has a low modulus of elasticity, which causes gentle force on the tooth or group of teeth. Gentle force means less stresses in the periodontal ligament (tissue). This is especially important for the patient’s well-being, as excessive rigidity of the material may cause damage to tissue [5].

When the orthodontist inserted the wire into the bracket attached to the separate tooth, he deformed the wire complexly, which induced a multi-axial stress state in the SMA NiTi. Namely, SMA NiTi has at different loading a variety of curves of superelasticity. It has different values of stress for the start (point B) and end (point C) of the phase transformation, as well as the length of the transformation plateau [6].

The aim of this work was from the experimental aspect to ascertain a value for the start of transformation at uni-axial tensile loading and determination of this value at compression and shear loading. For this research we used six commercially available NiTi orthodontic wires. To determine the value for the start of the transformation we used the theoretical calculation, because it is very difficult to make compressive and shear tests on small specimen such as orthodontic wires.

2. Materials and methods

2.1. Materials

For our investigation we used six commercially available orthodontic wires. The information necessary for our calculation we obtained from literature [7]. The values of stress for the start of transformation are given in Table 1 for all six wires. The specimen used and the tensile testing machine are shown in Figure 2. From the literature [7] it can be seen that the temperature of the samples where they have a fully austenitic phase is from 14 to 27 °C. All samples show a typical super elastic behaviour. The values for stress at the start of transformations for analysing orthodontic wires are very different and are in the range from 210 to 493 MPa.
Table 1. Values of tensile stress at the start of phase transformation taken from the uni-axial tensile test.

<table>
<thead>
<tr>
<th>Number of wire</th>
<th>Tensile stress $\sigma_{SM}^t$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>212</td>
</tr>
<tr>
<td>2</td>
<td>237</td>
</tr>
<tr>
<td>3</td>
<td>335</td>
</tr>
<tr>
<td>4</td>
<td>451</td>
</tr>
<tr>
<td>5</td>
<td>210</td>
</tr>
<tr>
<td>6</td>
<td>493</td>
</tr>
</tbody>
</table>

2.2. Theoretical calculation

For SMA NiTi most studies of the mechanical behaviour are performed on the uniaxial tensile tests. Tests showed asymmetry between tensile and compressive load in NiTi alloys [8]. Manach and Favier [9] are explaining that the Von Mises yield criterion assumptions, which are adopted typically in establishing tensorial constitutive equations, are not always valid for the simple shear test for NiTi alloys. Orgeas and Favier [10] in their study conducted investigations into the impact of different modes of deformation, which include: tension, compression and simple shear on the behaviour of the stress-induced martensite in the equatomic NiTi alloy. They used yield criterion proposed by Stutz which is expressed with the following relation:

$$Q_{\sigma_{cri}} = Q_{\sigma_{cri0}} / \left[1 + \gamma_0 \cdot \cos(3 \cdot \varphi_0)\right]^{n_0} \quad (1)$$

Parameters $Q_{\sigma_{cri0}}$, $\gamma_0$ and $n_0$ are given from experimental analysis. By solving system (1) we give shear (2) ($\varphi_0 = \pi/6$), tension (3) ($\varphi_0 = 0$) and compression (4) ($\varphi_0 = \pi$) stresses.

$$\tau_{cri} = \frac{Q_{\sigma_{cri0}}}{\sqrt{2}} \quad (2)$$

$$\sigma_{cri} = \frac{Q_{\sigma_{cri0}}}{(1+\gamma_0)^{n_0}} \quad (3)$$

$$\sigma_{cric} = \frac{Q_{\sigma_{cri0}}}{(1-\gamma_0)^{n_0}} \quad (4)$$

From the study [10] to consider, the parameter $\gamma_0$ and $n_0$ constant value 0.9 and 0.1. These experimentally determined constants give suitably results.

3. Results and discussion

Using data from the tensile test we calculated $Q_{\sigma_{cri0}}$ for all of six wires. These values were then taken into account in the calculation of the compressive and shear stress required for the start of transformation. In Table 2 are collected the calculated stresses to start transformation for the compressive $\sigma_{SMc}$ and shear $\tau_{SMs}$ loading. We can see that the stress for the start of phase transformation in the compressive loading is higher than the other two ways of loading. At shear loading we can see that the transformation starts at the least stress value for the separate orthodontic wire SMA NiTi.

Table 2. Calculated values for stress at the start of phase transformation under compression loading $\sigma_{SMc}$ and shear loading $\tau_{SMs}$.

<table>
<thead>
<tr>
<th>Number of wire</th>
<th>Tensile stress $\sigma_{SM}^t$ [MPa]</th>
<th>Compression stress $\sigma_{SMc}^t$ [MPa]</th>
<th>Shear stress $\tau_{SMs}^t$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>212</td>
<td>284</td>
<td>142</td>
</tr>
<tr>
<td>2</td>
<td>237</td>
<td>317</td>
<td>159</td>
</tr>
<tr>
<td>3</td>
<td>335</td>
<td>448</td>
<td>224</td>
</tr>
<tr>
<td>4</td>
<td>451</td>
<td>604</td>
<td>303</td>
</tr>
<tr>
<td>5</td>
<td>210</td>
<td>281</td>
<td>141</td>
</tr>
<tr>
<td>6</td>
<td>493</td>
<td>670</td>
<td>331</td>
</tr>
</tbody>
</table>

Figure 3 shows a polar representation of the stress tensor in the deviatoric stress plane with the used equation (1). Such kind of yield surface gives good results for isotropic polycrystalline NiTi in tension, compression and shear. As we can see, we have very different values for stresses at the start of the phase transformation. In the oral environment (at approx.37 °C), these values were slightly higher.

Figure 3. Comparison of tensile, compression and shear stress for different orthodontic wires.
From these data we can see that the first transformation causes the shear stresses. In orthodontic practice they appear often besides bending torsion stresses. In the case of torsion shear stress occurs before the transformation as in other cases. In our analysed wires it can be seen that we have a different range of stress values at the start of the phase transformation to martensite. The values for the tensile stress are in the range from 210 to approximately 500 MPa, obtained experimentally. The calculated values for the compression are between 281 to 670 MPa. In the case of shear, these values are from 141 to 331 MPa.

The results showed that these wires have very different values of stresses at the start of phase transformations. During orthodontic treatment we want to exploit the superelasticity plateau, because in this area is the smallest wire stiffness and, consequently, the result is a reduced force on the tooth. These data are particularly important for the orthodontist so that he can assess where the stiffness of the wire is smallest and the superelasticity plateau easier to achieve by deformation of the wire.

Conclusions

In this study were investigated the mechanical behaviour of NiTi SMA in different modes of loading such as tension, compression and shear.

Experimentally determined and theoretically calculated stresses were performed on six commercially available orthodontic wires. In this study we see from the use of any commercial wires that they have very different stress values at the start of the plateau.

Theoretical calculations give us sufficiently good approximation to the actual values, since they are based on experimental research and since most of the components were loaded with a combination of loads. Therefore, it is important to understand and to know the behaviour of the SMA NiTi, the asymmetry between tensile and compressive load and the behaviour under shear load.

Acknowledgments

This article has been supported by EUREKA Programme ORTO-NITI E!6788 within the framework of the Ministry of Higher Education, Science and Technology of the Republic of Slovenia and Programme for Young Researcher within the framework of the Slovenian Research Agency.

References